Appendix A – Model Structure

Figure A1: Model Structure

Figure A1 illustrates the overall model structure.

- Figure A1 shows the enhanced four stage structure of the model by modelled time period. It has the traditional Trip Generation, Mode Choice, Destination Choice and Assignment, enhanced by Park and Ride Site Choice Modelling

- The required data inputs to the model in forecasting mode are a set of trip ends as produced by the trip end model, and Road and public transport networks along with any model parameters which change through time (eg values of time, vehicle operating costs).

- The model is segmented by the following person household types:

    - No Car Households;
    - 1 Car 1 Adult Households;
    - 1 Car 2 Adult Households; and
    - 2+ Car Households.

- It is also segmented by the following six journey purposes:

    - home-based work;
    - home-based employer's business;
    - home-based other;
    - home-based education;
    - non-home-based employer's business; and
    - non-home-based other.

- The model is applied as an incremental model with the absolute forecasts produced representing an intermediate step in applying the model incrementally. The incremental approach is implemented in practice by using the model to calculate growth factors at a zone to zone cell level and to apply these to the Road and public transport assignment matrices.
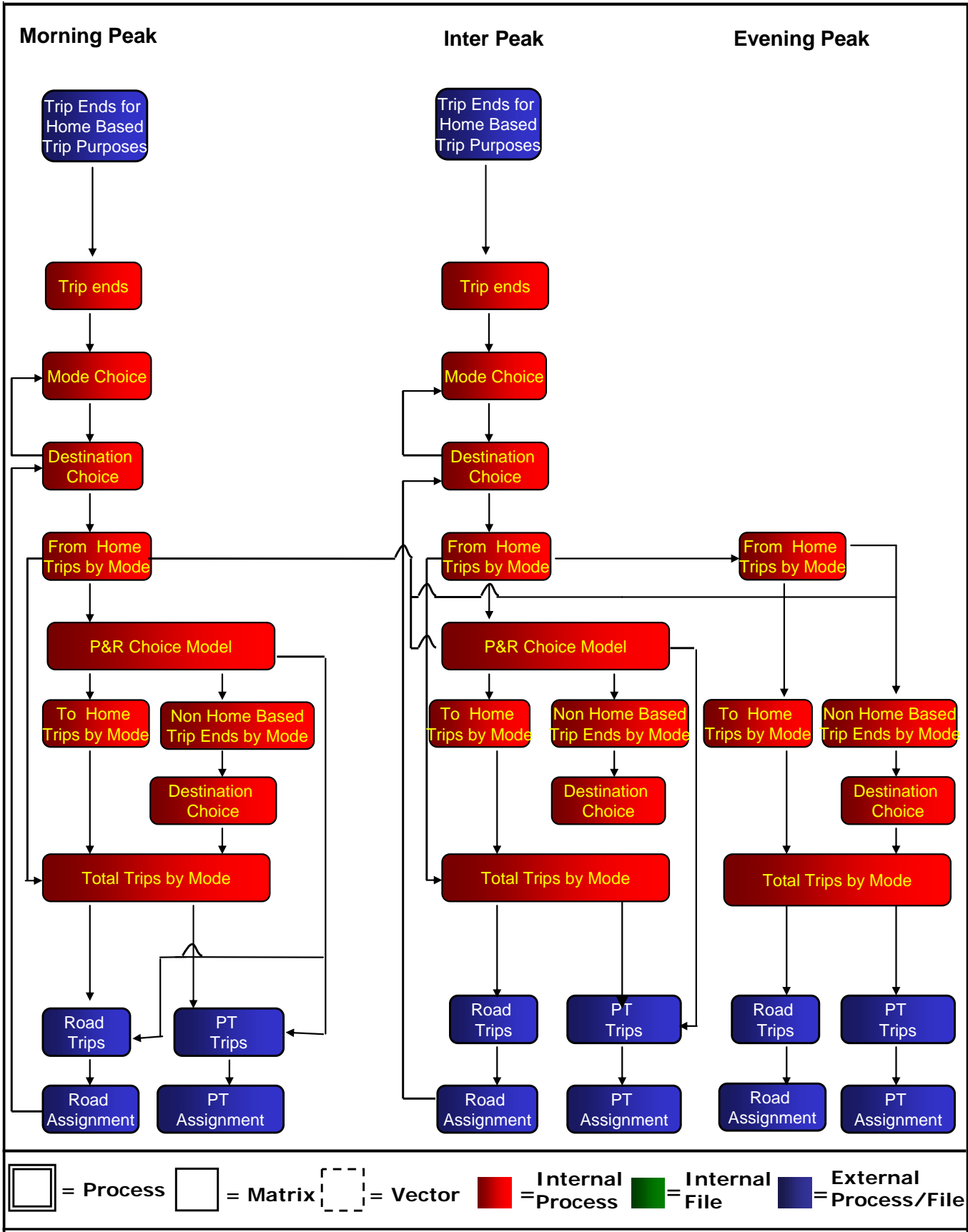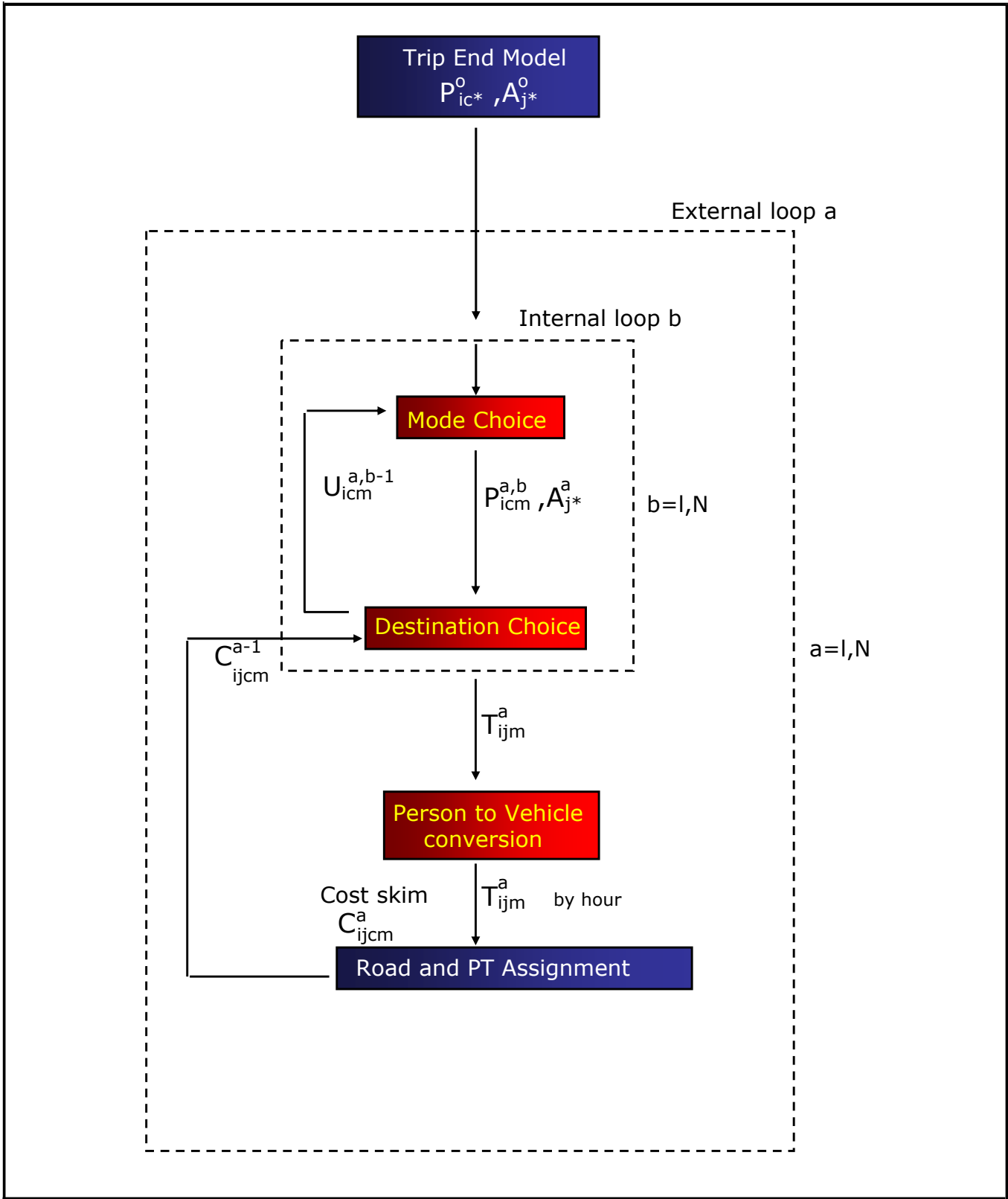
Figure A1: Time Period Demand Model

Figure A2: Iterative Demand/Supply Balancing

- Figure A2 shows the balancing of the supply and demand parts of the model. The process consists of a feedback of Generalised Costs from the assignment models to the distribution and mode split elements of the demand model.

- Two loops are shown in this figure, the inner loop 'b', which is the loop between the mode and destination choice elements of the model and the outer loop 'a', which is the loop between the demand model and the assignment models.

- The Public Transport costs are set after one external loop of the demand model. However, if crowding effects are considered sufficient to cause large changes in costs, it can be run on every external loop. The Road Assignment is carried out for each outer loop.

Trip End Model
P^o_{ic*} , A^o_{j*}

External loop a

Internal loop b

Mode Choice

$U^{a,b-1}_{icm}$

$P^{a,b}_{icm}$, A^a_{j*}

b=l,N

Destination Choice

C^{a-1}_{ijcm}

a=l,N

T^a_{ijm}

Person to Vehicle conversion

Cost skim
C^a_{ijcm}

T^a_{ijm} by hour

Road and PT Assignment

**Key:** C^a_{ijcm} - generalised cost skim for cell ij, car availability c, mode m, external loop a

$U^{a,b}_{icm}$ - logsum composite utility for zone i, car availability c, external loop a, internal loop b

Internal = Process

Internal = File

External = Process/File

Figure A2: Iterative Demand/Supply Balancing

Figure A3: Destination Choice

- Figure A3 shows the destination choice model for from-home purposes, which takes as inputs:

    - trip productions for car and public transport by each car availability segment;
    - trip attractions/attraction factors for all modes and car availability types combined; and
    - generalised costs of travel by car and by public transport.

- These inputs are used to create matrices of person trips, separately by time period and trip purpose, for from-home trips and non-home-based trips. The process is carried out at a zonal level.

- The process is a traditional gravity model process applied in a doubly constrained manner for from-home commute trips and non-home-based trips, and singly constrained for other from-home purposes. There are separate sensitivity parameters for each trip purpose/mode/car availability combination.

- The spread parameters are α_1 and α_2. There are also Intra-Zonal factors α_0, which produce intra-zonal trips in the matrix

- The outputs of this process are person trip matrices by time period, trip purpose/mode/car availability at zonal level, which are the inputs to the person to vehicle conversion process.
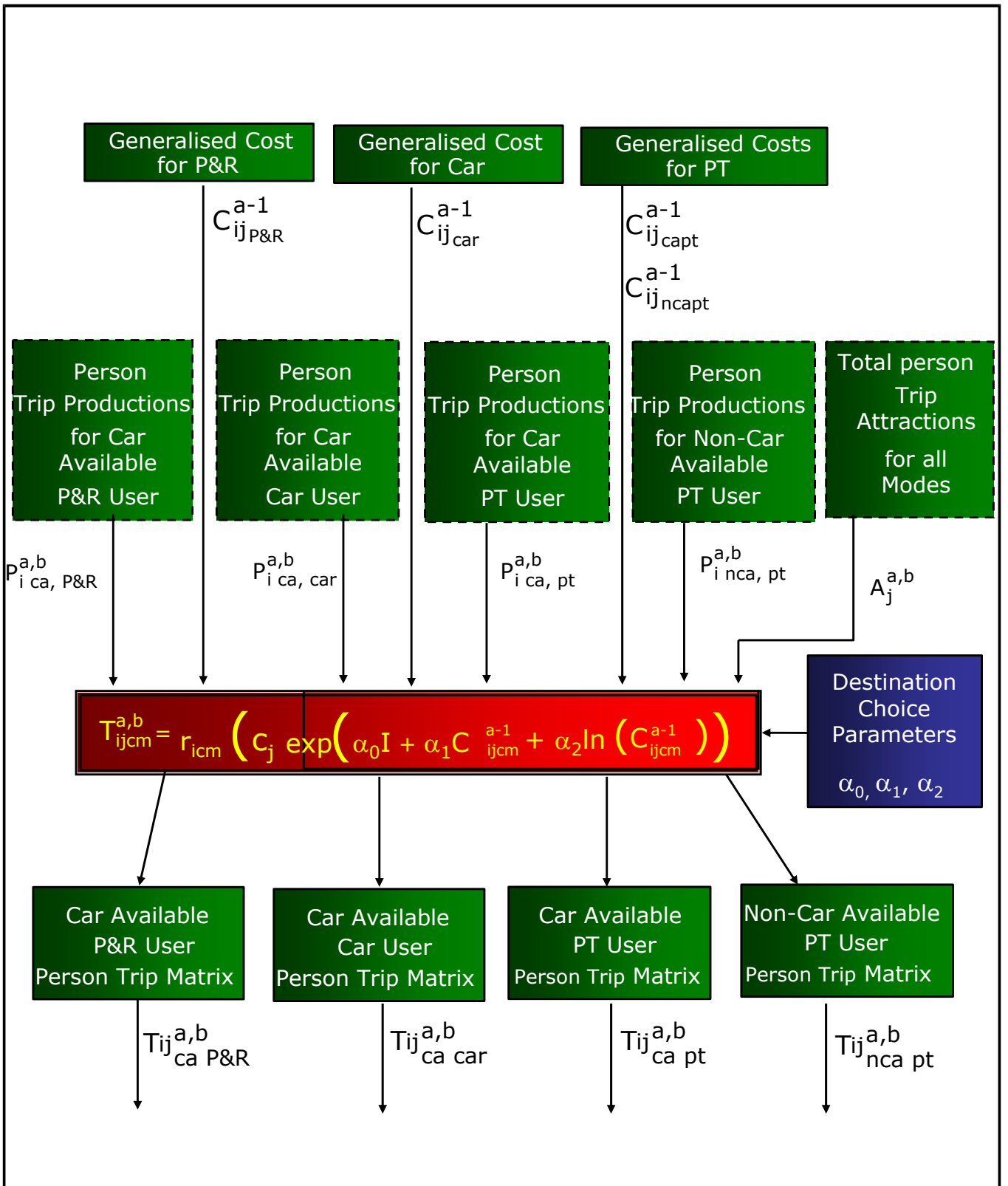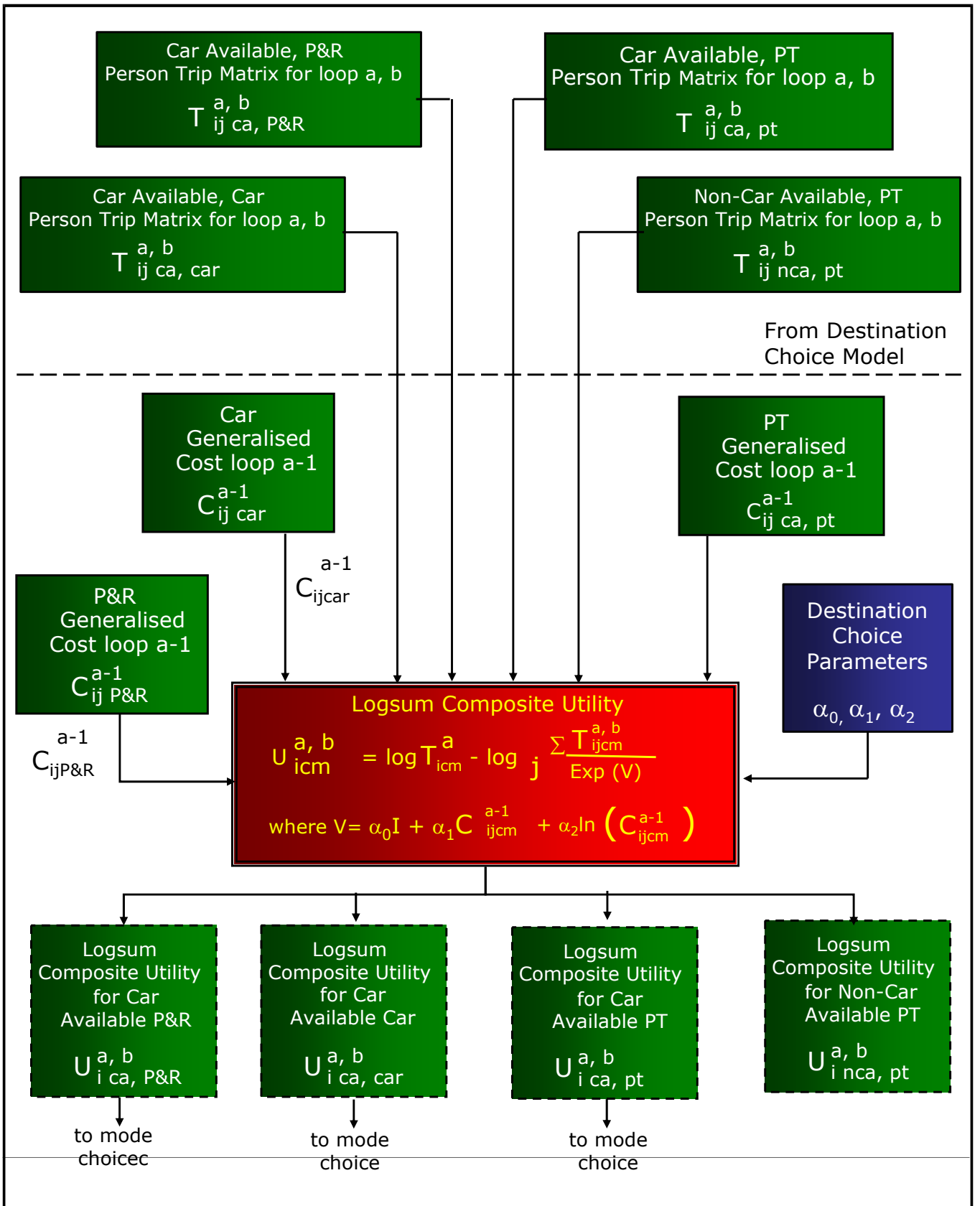
Figure A3: Destination Choice

Figure A4: Calculation of Logsum Composite Utility for Input to Mode Choice

- This process follows the destination choice process and calculates logsum composite utilities at a zonal level for input to the mode choice model.

- The process takes the matrices generated by the destination choice model along with the generalised costs by mode used in the destination choice model, and calculates a logsum composite utility for each zone.

- The process is conducted separately for each from-home trip purpose and car available segment. In the case of car available persons the output utilities are used as input to the mode choice process. For non-car available there is no mode choice.

- The sensitivity parameters (α_0, α_1, α_2) shown in this figure are the same sensitivity parameters used in the destination choice process.
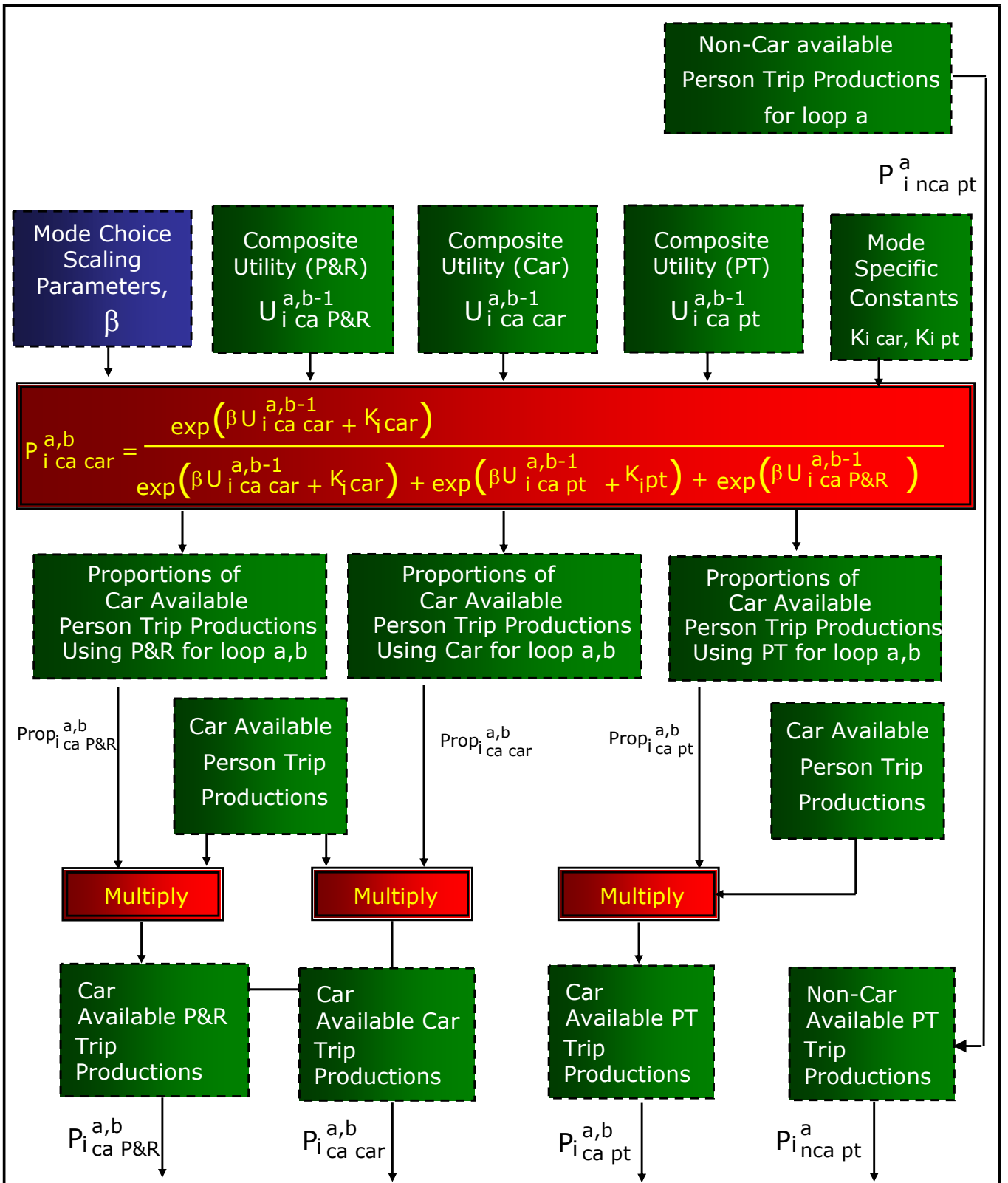
Figure A4: Calculation of Logsum Composite Utility for input to Mode Choice

The following describes the content within the figure:

Car Available, P&R
Person Trip Matrix for loop a, b
$T_{ij\ ca,\ P\&R}^{a,\ b}$

Car Available, PT
Person Trip Matrix for loop a, b
$T_{ij\ ca,\ pt}^{a,\ b}$

Car Available, Car
Person Trip Matrix for loop a, b
$T_{ij\ ca,\ car}^{a,\ b}$

Non-Car Available, PT
Person Trip Matrix for loop a, b
$T_{ij\ nca,\ pt}^{a,\ b}$

From Destination Choice Model

Car Generalised Cost loop a-1
$C_{ij\ car}^{a-1}$

PT Generalised Cost loop a-1
$C_{ij\ ca,\ pt}^{a-1}$

C_{ijcar}^{a-1}

P&R Generalised Cost loop a-1
$C_{ij\ P\&R}^{a-1}$

$C_{ijP\&R}^{a-1}$

Destination Choice Parameters
$\alpha_0,\ \alpha_1,\ \alpha_2$

Logsum Composite Utility

$$U_{icm}^{a,\ b} = \log T_{icm}^{a} - \log \sum_{j} \frac{T_{ijcm}^{a,\ b}}{Exp\ (V)}$$

$$\text{where } V = \alpha_0 I + \alpha_1 C_{ijcm}^{a-1} + \alpha_2 \ln \left(C_{ijcm}^{a-1} \right)$$

Logsum Composite Utility for Car Available P&R
$U_{i\ ca,\ P\&R}^{a,\ b}$

Logsum Composite Utility for Car Available Car
$U_{i\ ca,\ car}^{a,\ b}$

Logsum Composite Utility for Car Available PT
$U_{i\ ca,\ pt}^{a,\ b}$

Logsum Composite Utility for Non-Car Available PT
$U_{i\ nca,\ pt}^{a,\ b}$

to mode choicec

to mode choice

to mode choice

☐ = Process   ☐ = Matrix   ⌐ ¬ = Vector   ■ = Internal Process   ■ = Internal File   ■ = External Process/File

Figure A5: Mode Choice

- Figure A5 shows the mode choice process. It is undertaken at the trip end level due to the position of mode choice in the model hierarchy. It takes as inputs:

    - person trip productions by time period for from-home purposes; and
    - logsum composite utilities calculated using the destination choice model.

- The process is carried out for car available persons only; persons from non-car available households are assumed to be captive to public transport.

- The calculation of the logsum composite utilities is undertaken using the process described in Figure A4.

- There are two types of parameter input:

    - The scaling factors (β) which control the sensitivity of the mode choice process.

    - The mode specific constants (for car mode only) which ensure that the base mode choice proportions in the model match the base data at a zonal level.

- There are separate scaling parameters for each trip purpose.

- The outputs of the process are vectors of trip productions which are then input to the destination choice process which is shown in Figure A3.
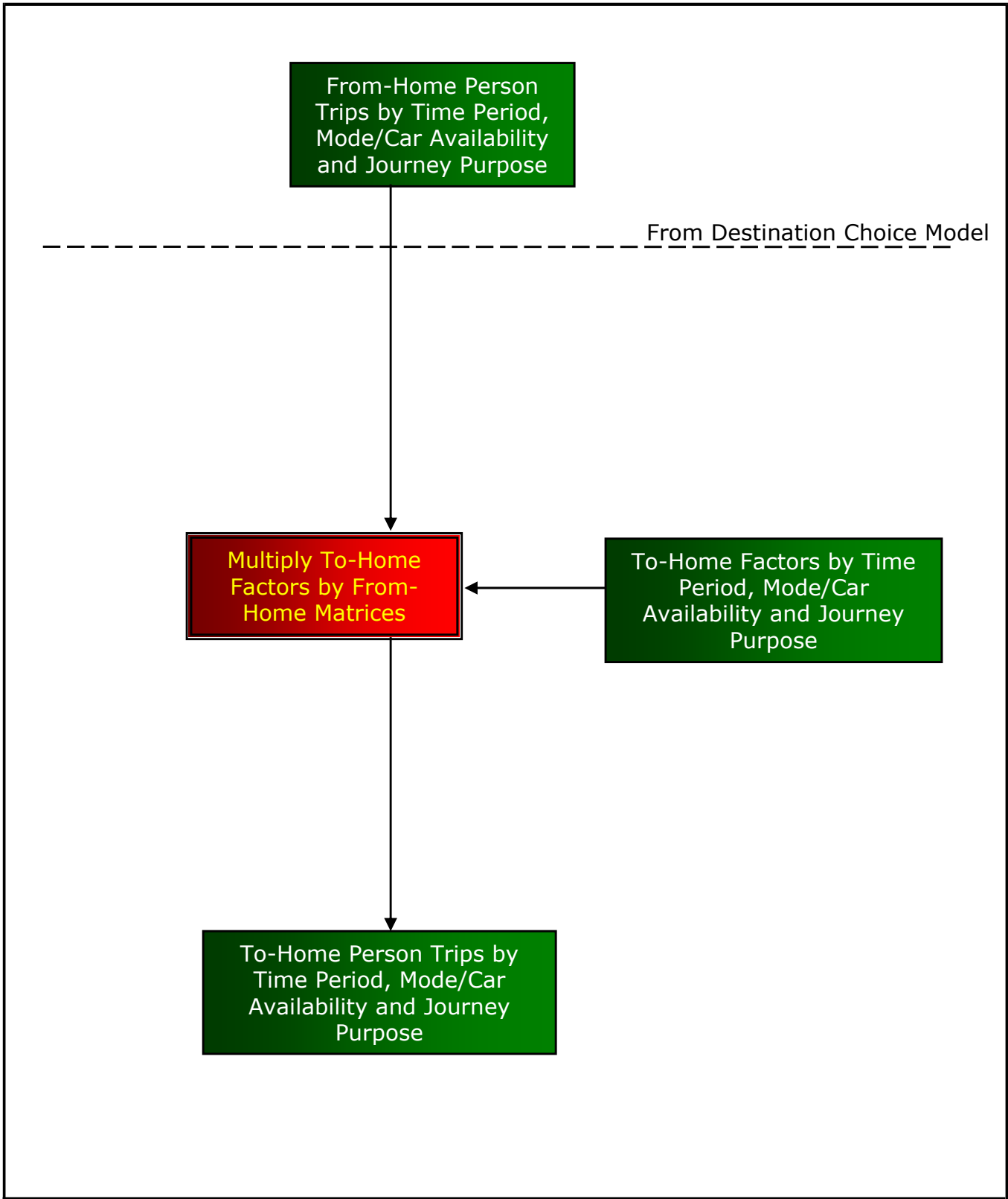
Non-Car available
Person Trip Productions
for loop a

$P^a_{i\ nca\ pt}$

Mode Choice
Scaling
Parameters,
β

Composite
Utility (P&R)
$U^{a,b-1}_{i\ ca\ P\&R}$

Composite
Utility (Car)
$U^{a,b-1}_{i\ ca\ car}$

Composite
Utility (PT)
$U^{a,b-1}_{i\ ca\ pt}$

Mode
Specific
Constants
$K_{i\ car},\ K_{i\ pt}$

$$P^{a,b}_{i\ ca\ car} = \frac{\exp\left(\beta U^{a,b-1}_{i\ ca\ car} + K_{i\ car}\right)}{\exp\left(\beta U^{a,b-1}_{i\ ca\ car} + K_{i\ car}\right) + \exp\left(\beta U^{a,b-1}_{i\ ca\ pt} + K_{i\ pt}\right) + \exp\left(\beta U^{a,b-1}_{i\ ca\ P\&R}\right)}$$

Proportions of
Car Available
Person Trip Productions
Using P&R for loop a,b

Proportions of
Car Available
Person Trip Productions
Using Car for loop a,b

Proportions of
Car Available
Person Trip Productions
Using PT for loop a,b

$Prop^{a,b}_{i\ ca\ P\&R}$

Car Available
Person Trip
Productions

$Prop^{a,b}_{i\ ca\ car}$

$Prop^{a,b}_{i\ ca\ pt}$

Car Available
Person Trip
Productions

Multiply

Multiply

Multiply

Car
Available P&R
Trip
Productions

Car
Available Car
Trip
Productions

Car
Available PT
Trip
Productions

Non-Car
Available PT
Trip
Productions

$P_{i\ ca\ P\&R}^{a,b}$

$P_{i\ ca\ car}^{a,b}$

$P_{i\ ca\ pt}^{a,b}$

$P_{i\ nca\ pt}^{a}$

☐ = Process   ☐ = Matrix   ⌐ ¬ = Vector   ■ = Internal Process   ■ = Internal File   ■ = External Process/File

Figure A5: Mode Choice

Figure A6: Creation of To-Home Trips

- This model produces the matrices of to-home trips by mode, car availability, time period and purpose. It takes as input:

    - matrices of from-home trips, by mode, car availability, journey purpose and time period, as produced by the destination choice models; and
    - to-home factors derived from analysis of the Scottish Household Survey.

- The to-home factors are multiplied by the transposed from-home matrices at a cellular level to give to-home matrices by mode, car availability, journey purpose and time period.
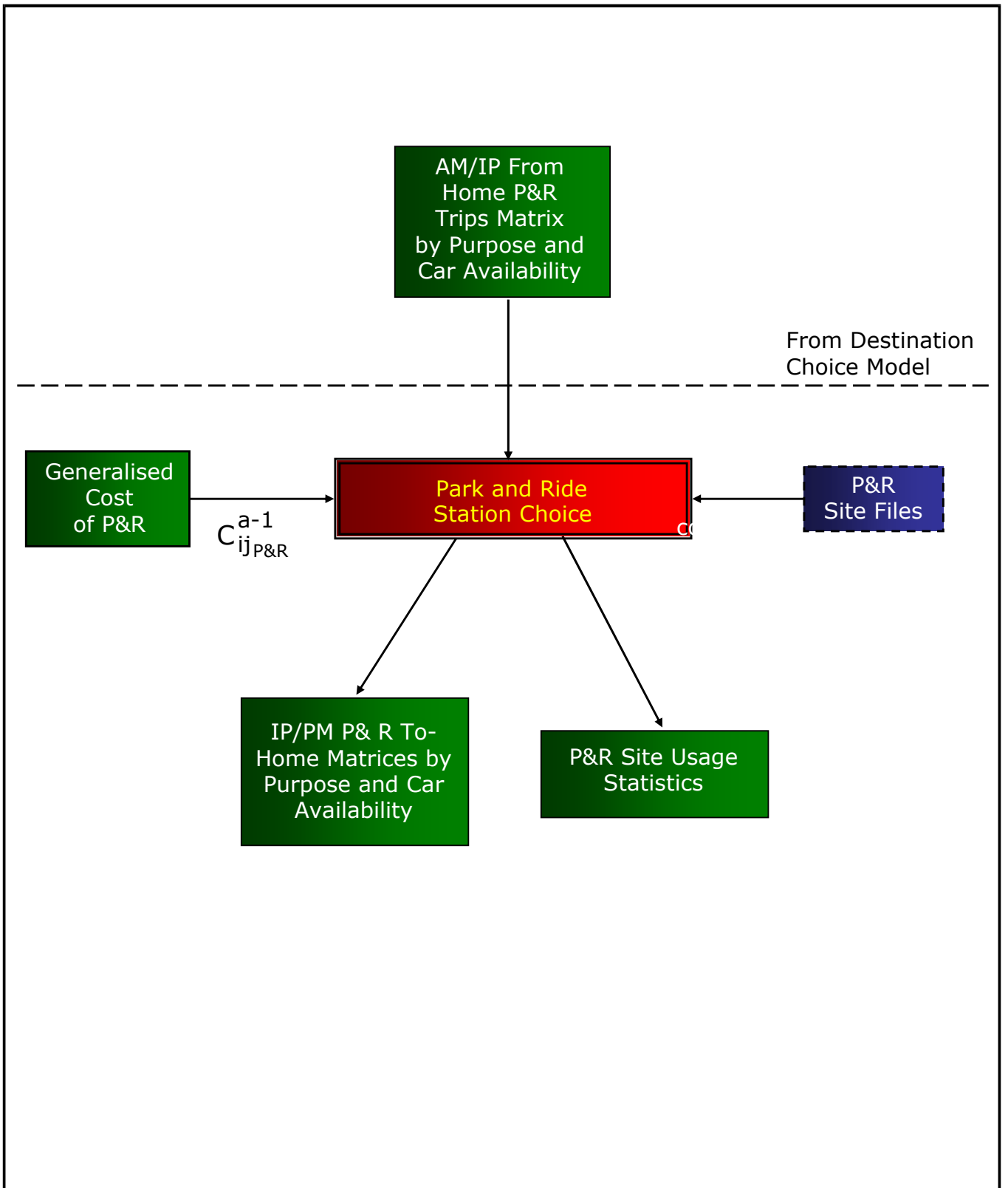
From-Home Person
Trips by Time Period,
Mode/Car Availability
and Journey Purpose

From Destination Choice Model

Multiply To-Home
Factors by From-
Home Matrices

To-Home Factors by Time
Period, Mode/Car
Availability and Journey
Purpose

To-Home Person Trips by
Time Period, Mode/Car
Availability and Journey
Purpose

☐ = Process   ☐ = Matrix   ⌐⌐ = Vector   ■ =Internal Process   ■ =Internal File   ■ =External Process/File

Figure A6: Creation of To-Home Matrices

Figure A7: Park and Ride Station Choice

- Park and Ride station choice is applied to the AM and Inter-Peak From-Home Park and ride trips, which are output for each purpose by the mode/destination choice model.

- The choice model takes as input:
    - The generalised cost of P&R;
    - Park and Ride Site Files; and
    - Park and Ride Trip Matrices.

- The station choice model then attributes all the Park and Ride trips amongst all the sites available, based on the attributes of the sites.

AM/IP From
Home P&R
Trips Matrix
by Purpose and
Car Availability

From Destination
Choice Model

Generalised
Cost
of P&R

$C_{ij_{P\&R}}^{a-1}$

Park and Ride
Station Choice

P&R
Site Files

IP/PM P& R To-
Home Matrices by
Purpose and Car
Availability

P&R Site Usage
Statistics

= Process   = Matrix   = Vector   = Internal Process   = Internal File   = External Process/File

Figure A7: Park and Ride Station Choice

Figure A8: Trip End Model for Non-Home-Based Purposes

- This model produces the trip productions and attractions by mode, time period and purpose for non-home-based trips. It takes as input:

    - trip attractions for From-Home trips, by mode, car availability, journey purpose and time period, as produced by the destination choice models;
    - trip productions for To-Home trips, by mode, car availability, journey purpose and time period;
    - non-home-based production trip rate factors derived from analysis of the Scottish Household Survey; and
    - non-home-based attraction trip rate factors derived from analysis of the Scottish Household Survey.

- The production trip rate factors are multiplied by the From-Home trip attractions to give the Non Home Based trip productions by mode, car availability, journey purpose and time period.

- The attraction trip rate factors are multiplied by the To-Home trip productions to give the Non Home Based trip attractions by mode, car availability, journey purpose and time period.
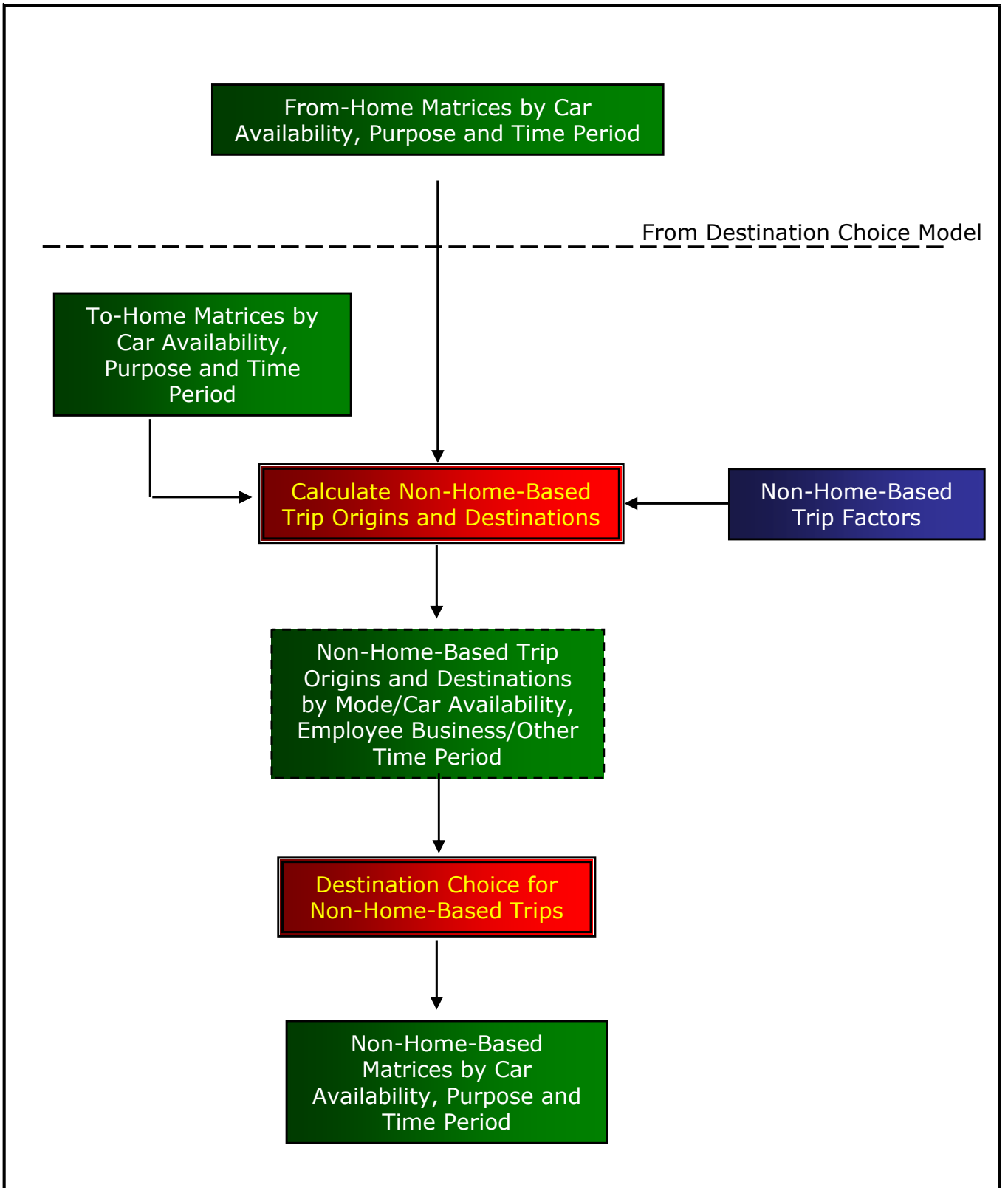
Figure A8: Trip End Model for Non-Home-Based Purposes

Figure A9: Person to Vehicle Conversion for Car Trips

- This process takes as input the time period matrices which are output by the processes shown in Figures 3, 6, 7 and 8 as well as Education add-in's, and applies vehicle occupancy factors to convert from person trips to vehicle trips.

- The process is carried out separately for each trip purpose and there are separate vehicle occupancy factors for each time period/trip purpose combination.
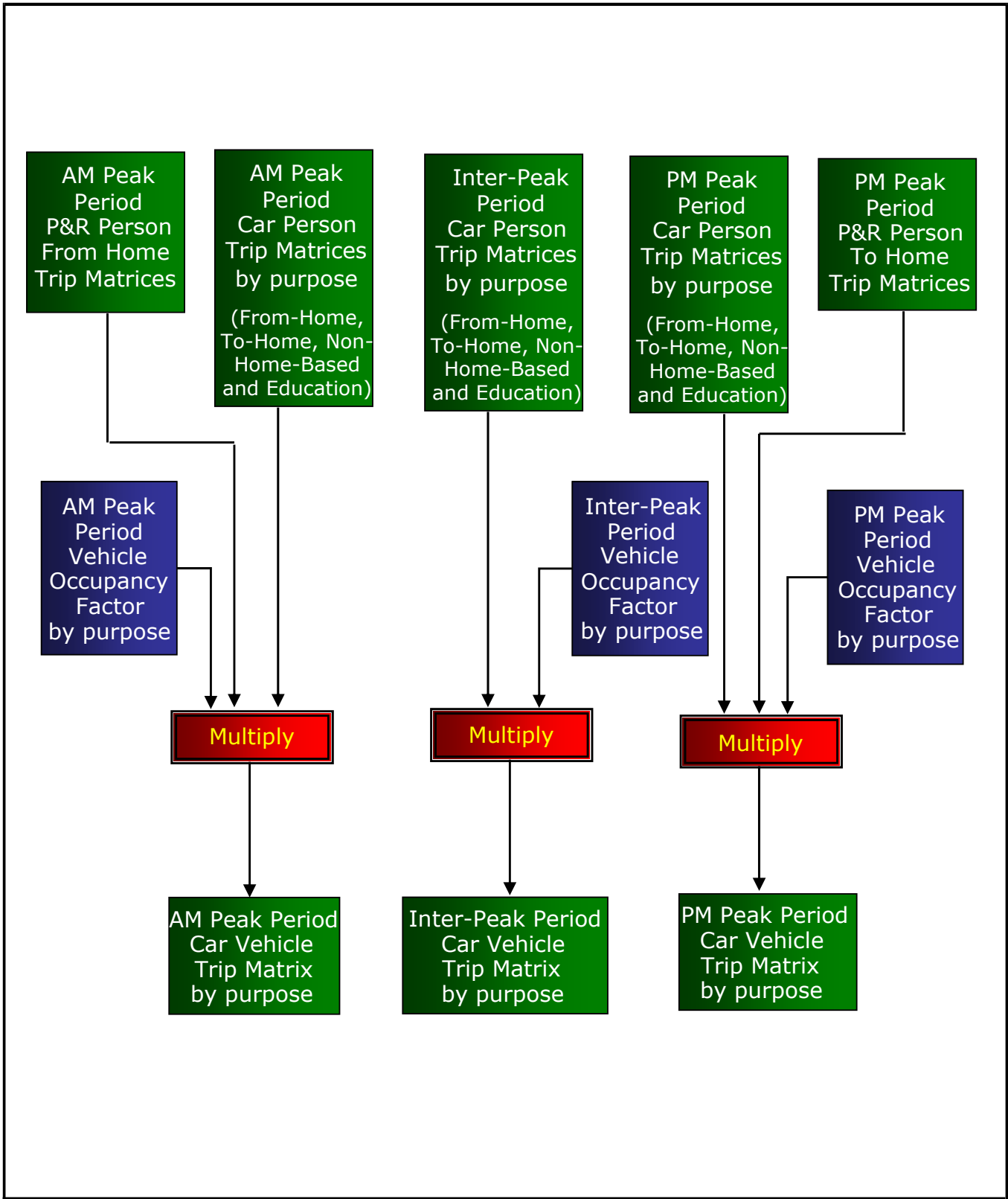
Figure A9: Person to Vehicle Conversion for Car Trips

Figure A10: Hourly Matrix Creation for Car and Public Transport

- This process converts the time period matrices by vehicle or person, as appropriate, into hourly matrices.

- The process is carried out for all trip purposes. The Inter-Peak and PM Peak factors will generally remain the same in all forecasting situations.
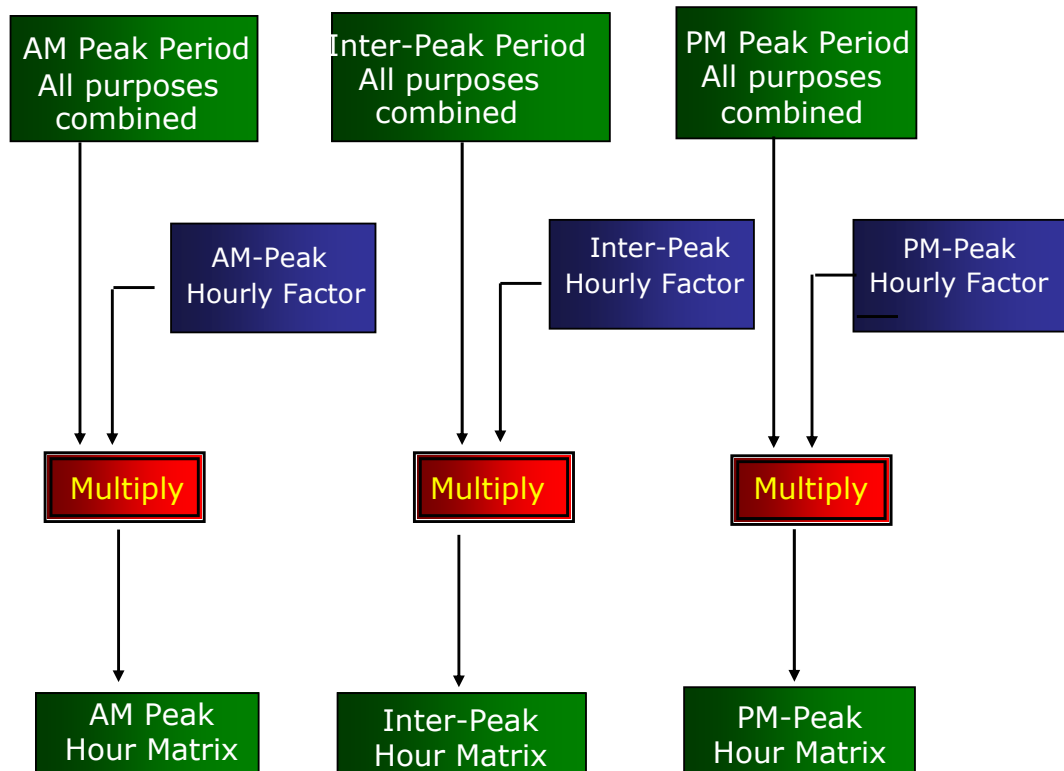
Figure A10: Hourly Matrix Creation for car and public transport

Figure A11: Incremental Adjustments

- The incremental adjustment process ensures that the model functions in an incremental manner. The incremental adjustment factors are applied to Origin-Destination matrices (separately by hour and mode), to generate the final hourly assignment matrices at zonal level. These are applied in both an additive and multiplicative fashion.
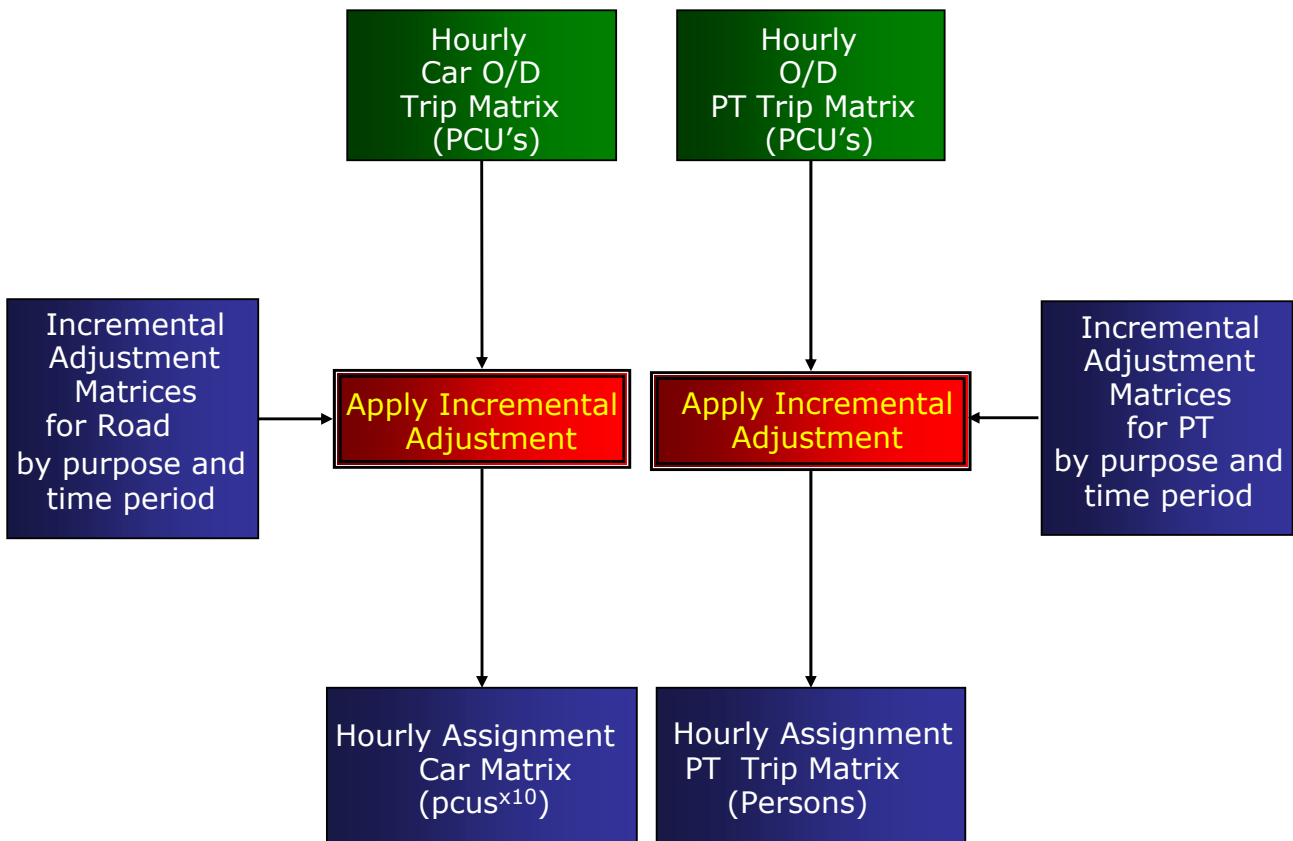
Figure A11: Incremental Adjustment

Figure A12: Road Assignment

- This is a conventional capacity restrained equilibrium multi-class assignment procedure. It is operated using the TRIPS program suite. There are five user classes:

    1. Car In-Work;
    2. Car Commute;
    3. Car Other;
    4. Light goods vehicles; and
    5. Heavy goods vehicles.

- The cost skims are created at a zonal level from the assignment model. Parking charges are added into the cost skims where appropriate.
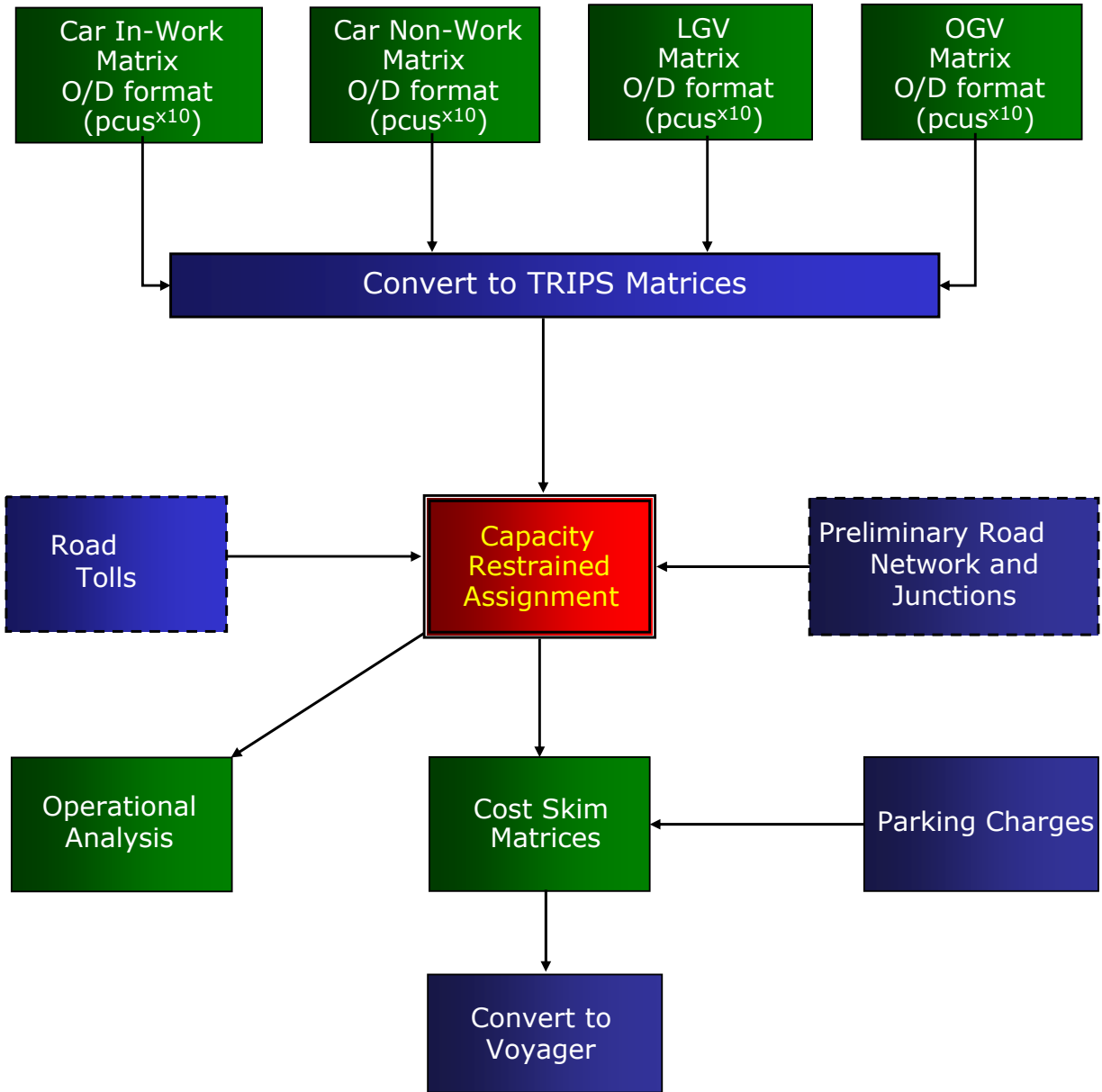
Figure A12: Road Assignment

Figure A13: Public Transport Passenger Assignment

- This is a standard PT assignment procedure, with sub-mode choice, undertaken in TRIPS. The PT model excludes the effects of passenger crowding. It is standard to skim the PT costs only on the first loop of the demand model. However, considered necessary, the PT model can be skimmed on every loop of the model.
- There are three user classes:

    1.   *In-Work;*
    2.   *Commute; and*
    3.   *Other.*

Hourly PT Trip matrix (persons, zones)

Junction Delays from Road Network

Assignment (Including PT Sub Mode Split)

PT Network Service and Line Description

PT Cost Skims

Operational Analysis

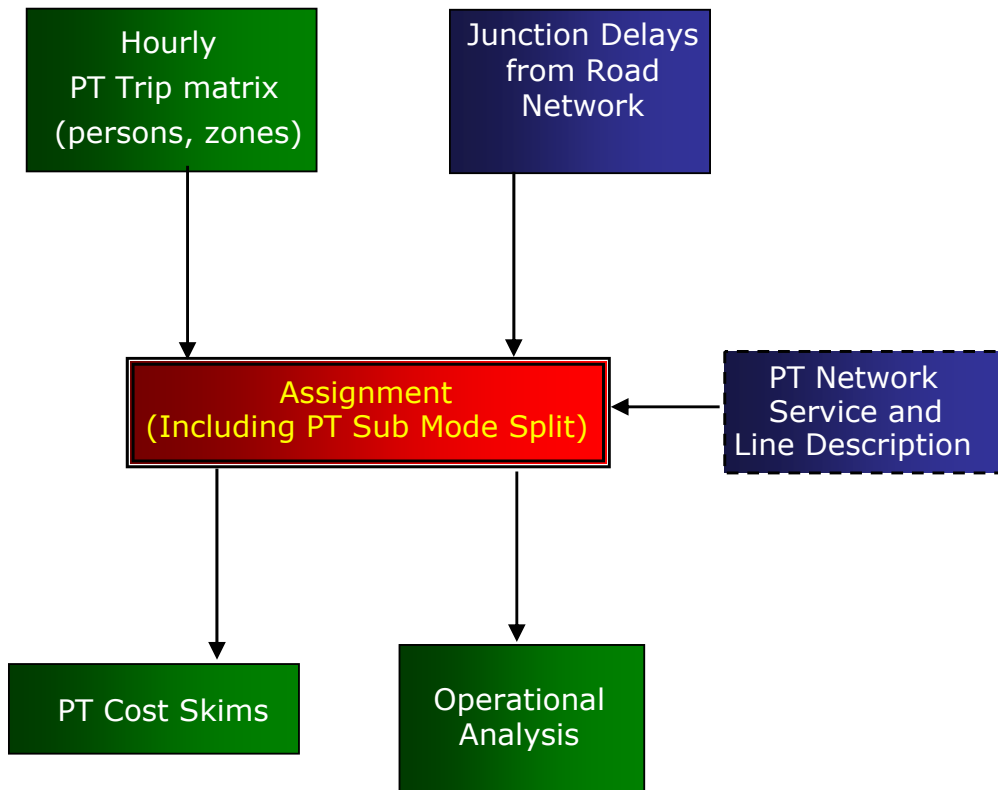☐ = Process ☐ = Matrix ⌐⌐ = Vector ■ = Internal Process ■ = Internal File ■ = External Process/File

Figure A13: Public Transport Passenger Assignment

Appendix B – SEStran Regional Model Zones

F SEStran Regional Model Zone Structure

The SEStran Regional Model Zones are a disaggregated version of the TMfS:07 National Model Zones. Table B1 below shows the full set of internal zones (zones 1 to 335) along with their TMfS:07 equivalents.

The zones are Census Output Area compatible and, like the National Model, there is at most one train station per zone. The zones are also consistent with Local Authority boundaries.

Table B1 SEStran Regional Model Zones

SRM Zone	TMfS:07 Zone	Local Authority
1	75	City of Edinburgh
2	72	City of Edinburgh
3	72	City of Edinburgh
4	72	City of Edinburgh
5	70	City of Edinburgh
6	70	City of Edinburgh
7	70	City of Edinburgh
8	73	City of Edinburgh
9	73	City of Edinburgh
10	73	City of Edinburgh
11	73	City of Edinburgh
12	69	City of Edinburgh
13	73	City of Edinburgh
14	69	City of Edinburgh
15	70	City of Edinburgh
16	67	City of Edinburgh
17	74	City of Edinburgh
18	74	City of Edinburgh
19	68	City of Edinburgh
20	68	City of Edinburgh
21	709	City of Edinburgh
22	71	City of Edinburgh
23	65	City of Edinburgh

24	65	City of Edinburgh
25	65	City of Edinburgh
26	61	City of Edinburgh
27	63	City of Edinburgh
28	62	City of Edinburgh
29	62	City of Edinburgh
30	64	City of Edinburgh
31	69	City of Edinburgh
32	69	City of Edinburgh
33	69	City of Edinburgh
34	142	West Lothian
35	138	West Lothian
36	139	West Lothian
37	139	West Lothian
38	62	City of Edinburgh
39	62	City of Edinburgh
40	140	West Lothian
41	133	West Lothian
42	137	West Lothian
43	137	West Lothian
44	391	Falkirk
45	137	West Lothian
46	129	West Lothian
47	390	Falkirk
48	389	Falkirk
49	387	Falkirk
50	388	Falkirk
51	384	Falkirk
52	386	Falkirk
53	385	Falkirk
54	382	Falkirk
55	383	Falkirk
56	376	Falkirk
57	376	Falkirk
58	376	Falkirk

59	379	Falkirk
60	380	Falkirk
61	381	Falkirk
62	382	Falkirk
63	378	Falkirk
64	375	Falkirk
65	374	Falkirk
66	374	Falkirk
67	375	Falkirk
68	372	Falkirk
69	373	Falkirk
70	372	Falkirk
71	371	Falkirk
72	372	Falkirk
73	468	Stirling
74	468	Stirling
75	469	Stirling
76	468	Stirling
77	463	Stirling
78	463	Stirling
79	463	Stirling
80	463	Stirling
81	463	Stirling
82	463	Stirling
83	469	Stirling
84	469	Stirling
85	463	Stirling
86	466	Stirling
87	466	Stirling
88	466	Stirling
89	466	Stirling
90	466	Stirling
91	462	Stirling
92	462	Stirling
93	462	Stirling

94	464	Stirling
95	467	Stirling
96	467	Stirling
97	467	Stirling
98	470	Clackmannanshire
99	472	Clackmannanshire
100	472	Clackmannanshire
101	472	Clackmannanshire
102	472	Clackmannanshire
103	472	Clackmannanshire
104	472	Clackmannanshire
105	472	Clackmannanshire
106	470	Clackmannanshire
107	470	Clackmannanshire
108	470	Clackmannanshire
109	471	Clackmannanshire
110	473	Clackmannanshire
111	473	Clackmannanshire
112	473	Clackmannanshire
113	475	Clackmannanshire
114	471	Clackmannanshire
115	474	Clackmannanshire
116	475	Clackmannanshire
117	477	Fife
118	478	Fife
119	479	Fife
120	476	Clackmannanshire
121	476	Clackmannanshire
122	476	Clackmannanshire
123	533	Perth & Kinross
124	488	Fife
125	483	Fife
126	490	Fife
127	483	Fife
128	481	Fife

129	480	Fife
130	480	Fife
131	480	Fife
132	482	Fife
133	485	Fife
134	486	Fife
135	486	Fife
136	484	Fife
137	484	Fife
138	484	Fife
139	484	Fife
140	489	Fife
141	487	Fife
142	492	Fife
143	491	Fife
144	493	Fife
145	497	Fife
146	494	Fife
147	494	Fife
148	496	Fife
149	495	Fife
150	533	Perth & Kinross
151	499	Fife
152	497	Fife
153	503	Fife
154	501	Fife
155	499	Fife
156	500	Fife
157	508	Fife
158	512	Fife
159	509	Fife
160	502	Fife
161	504	Fife
162	510	Fife
163	510	Fife

164	511	Fife
165	505	Fife
166	507	Fife
167	507	Fife
168	525	Fife
169	524	Fife
170	522	Fife
171	519	Fife
172	518	Fife
173	516	Fife
174	515	Fife
175	506	Fife
176	513	Fife
177	517	Fife
178	521	Fife
179	523	Fife
180	520	Fife
181	514	Fife
182	498	Fife
183	541	Perth & Kinross
184	540	Perth & Kinross
185	530	Perth & Kinross
186	528	Perth & Kinross
187	465	Stirling
188	377	Falkirk
189	368	North Lanarkshire
190	119	West Lothian
191	369	North Lanarkshire
192	120	West Lothian
193	122	West Lothian
194	123	West Lothian
195	127	West Lothian
196	124	West Lothian
197	126	West Lothian
198	128	West Lothian

199	130	West Lothian
200	131	West Lothian
201	132	West Lothian
202	135	West Lothian
203	136	West Lothian
204	134	West Lothian
205	141	West Lothian
206	66	City of Edinburgh
207	51	Midlothian
208	76	City of Edinburgh
209	77	City of Edinburgh
210	83	City of Edinburgh
211	82	City of Edinburgh
212	80	City of Edinburgh
213	81	City of Edinburgh
214	78	City of Edinburgh
215	79	City of Edinburgh
216	87	City of Edinburgh
217	89	City of Edinburgh
218	92	City of Edinburgh
219	96	City of Edinburgh
220	86	City of Edinburgh
221	98	City of Edinburgh
222	94	City of Edinburgh
223	90	City of Edinburgh
224	84	City of Edinburgh
225	85	City of Edinburgh
226	93	City of Edinburgh
227	100	City of Edinburgh
228	91	City of Edinburgh
229	101	City of Edinburgh
230	88	City of Edinburgh
231	95	City of Edinburgh
232	110	City of Edinburgh
233	103	City of Edinburgh

234	106	City of Edinburgh
235	111	City of Edinburgh
236	113	City of Edinburgh
237	108	City of Edinburgh
238	105	City of Edinburgh
239	99	City of Edinburgh
240	97	City of Edinburgh
241	107	City of Edinburgh
242	102	City of Edinburgh
243	104	City of Edinburgh
244	109	City of Edinburgh
245	112	City of Edinburgh
246	115	City of Edinburgh
247	114	City of Edinburgh
248	116	City of Edinburgh
249	117	City of Edinburgh
250	118	City of Edinburgh
251	56	Midlothian
252	55	Midlothian
253	54	Midlothian
254	52	Midlothian
255	59	Midlothian
256	39	East Lothian
257	36	East Lothian
258	38	East Lothian
259	37	East Lothian
260	121	West Lothian
261	125	West Lothian
262	31	Borders
263	31	Borders
264	31	Borders
265	22	Borders
266	22	Borders
267	23	Borders
268	23	Borders

269	23	Borders
270	24	Borders
271	24	Borders
272	26	Borders
273	30	Borders
274	30	Borders
275	34	Borders
276	34	Borders
277	34	Borders
278	34	Borders
279	35	Borders
280	35	Borders
281	33	Borders
282	32	Borders
283	32	Borders
284	32	Borders
285	29	Borders
286	29	Borders
287	29	Borders
288	29	Borders
289	28	Borders
290	28	Borders
291	28	Borders
292	28	Borders
293	28	Borders
294	25	Borders
295	25	Borders
296	25	Borders
297	27	Borders
298	27	Borders
299	27	Borders
300	27	Borders
301	27	Borders
302	27	Borders
303	27	Borders

304	31	Borders
305	49	Borders
306	49	East Lothian
307	49	East Lothian
308	47	East Lothian
309	47	East Lothian
310	46	East Lothian
311	46	East Lothian
312	45	East Lothian
313	45	East Lothian
314	45	East Lothian
315	44	East Lothian
316	48	East Lothian
317	48	East Lothian
318	41	East Lothian
319	41	East Lothian
320	41	East Lothian
321	40	East Lothian
322	40	East Lothian
323	42	East Lothian
324	42	East Lothian
325	43	East Lothian
326	60	Midlothian
327	57	Midlothian
328	57	Midlothian
329	58	Midlothian
330	58	Midlothian
331	58	Midlothian
332	53	Midlothian
333	50	Midlothian
334	50	Midlothian
335	50	Midlothian

The number of zones in each Local Authority covered by the model is shown in Table B2.

Table B2 SEStran Regional Model – Number of Zones by Local Authority

Local Authority	Number of Zones
City of Edinburgh	79
Clackmannanshire	22
East Lothian	25
Falkirk	28
Fife	61
Midlothian	16
North Lanarkshire	2
Perthshire & Kinross	6
Stirling	26
The Borders	43
West Lothian	27

Zones 336 to 371 are the route zones. These route zones are described in table B3.

Table B3 SEStran Regional Model Route Zones

FRC Route Zone	Description
336	A701
337	A721
338	B7016
339	A721
340	A702
341	A708
342	A7
343	A68
344	A6105
345	A1
346	A697

347	Tay Bridge
348	A90
349	A989
350	Glasgow Road
351	A9
352	B8062
353	A822
354	B827
355	A820
356	B8075
357	A811
358	B818
359	A803
360	B816
361	A80
362	B803
363	A89
364	M8
365	B7066
366	B717
367	B7010
368	B7015
369	A706
370	A71
371	A70
336	A701
337	A721
338	B7016
339	A721

The final group of zones are 372-470 which have been left as Park and Ride Zones. In the base model only some of these have been used and they are listed in Appendix G.

Appendix C – Generalised Cost Parameters

Calculation of Base Year Demand Model Cost Coefficients

These parameters have been calculated following the guidance in the most up to date version of WebTAG unit 3.5.6 available at the time of the model development. Generalised Cost coefficients are calculated separately for all the assigned purposes (for both Highway and PT).

Highway Coefficients

Car Non-Work time generalised cost parameters were calculated in accordance with WebTAG, as follows:

Time Cost Parameter = 1

Distance cost parameter = (Fuel Cost + Vehicle Operating Cost)/VOT *(mins per kilometre)*; and

Toll cost parameter = 1/ VOT for tolling *(mins per pence)*

Where,

Fuel Cost = $consumption_{petrol}$ x $price_{petrol}$ x $Prop_{petrol}$ + $consumption_{diesel}$ x $price_{diesel}$ x $Prop_{diesel}$;

Consumption = $a + bV + cV^2 + dV^3$ (a, b, c and d are coefficients from WebTAG Table 13);

Vehicle Operating Cost = $a + b/V$ (a and b are coefficients from WebTAG Table 15)

Note: the appropriate growth factors have been applied to each step to bring the model values up to 2007, the fuel efficiency savings over time are also applied)

Note that for the purpose of the calculations the average speed in the modelled area has been calculated as 52 km/h.

The Generalised Cost coefficients for all possible forecast years are shown in table C1. To get the Highway coefficients into a format that the SATURN highway assignment model can read them in, they are all multiplied by the VOT/60.

PT Coefficients

For the PT Model the Values of Time used in the assignment are taken from WebTAG unit 3.5.6 tables 1 and 2. These are then growthed up to 2007 using the values in WebTAG Table 3.

It should be noted that Commute and Other purposes use the same value of time in assignment. This is a weighted average of the two Values of Time.

The Values of Time for all possible Modelled Years is shown in Table C1.

Table C1 Generalised Cost Coefficients by Year

Year	Car In Work			Car Non Work (Commuting)			Car Non Work (Other)			LGV			OGV			PT	
	Time	Distance	Tolls	Time	Distance	Tolls	Time	Distance	Tolls	Time	Distance	Tolls	Time	Distance	Tolls	NW	IW
2007	1	0.2336	0.0503	1	0.5342	0.1448	1	0.3902	0.1448	1	0.6824	0.0225	1	2.1988	0.0225	511.266	2157.901
2008	1	0.2285	0.0492	1	0.5242	0.1423	1	0.3836	0.1423	1	0.6685	0.0220	1	2.1598	0.0220	575.917	2430.775
2009	1	0.2197	0.0481	1	0.4946	0.1398	1	0.3627	0.1398	1	0.6399	0.0216	1	2.0561	0.0216	586.053	2473.556
2010	1	0.2130	0.0471	1	0.4752	0.1374	1	0.3490	0.1374	1	0.6190	0.0211	1	1.9835	0.0211	596.368	2517.091
2011	1	0.2075	0.0461	1	0.4619	0.1350	1	0.3399	0.1350	1	0.6067	0.0206	1	1.9434	0.0206	606.864	2561.392
2012	1	0.2027	0.0452	1	0.4499	0.1330	1	0.3316	0.1330	1	0.5962	0.0202	1	1.9089	0.0202	616.270	2601.093
2013	1	0.1980	0.0443	1	0.4382	0.1309	1	0.3236	0.1309	1	0.5859	0.0199	1	1.8750	0.0199	625.823	2641.410
2014	1	0.1934	0.0435	1	0.4268	0.1289	1	0.3158	0.1289	1	0.5757	0.0195	1	1.8418	0.0195	635.523	2682.352
2015	1	0.1889	0.0427	1	0.4157	0.1270	1	0.3081	0.1270	1	0.5658	0.0191	1	1.8092	0.0191	645.373	2723.929
2016	1	0.1844	0.0419	1	0.4041	0.1250	1	0.3001	0.1250	1	0.5560	0.0187	1	1.7771	0.0187	655.377	2766.149
2017	1	0.1800	0.0411	1	0.3929	0.1231	1	0.2923	0.1231	1	0.5463	0.0184	1	1.7456	0.0184	665.535	2809.025
2018	1	0.1757	0.0403	1	0.3820	0.1212	1	0.2847	0.1212	1	0.5368	0.0180	1	1.7146	0.0180	675.851	2852.565
2019	1	0.1716	0.0395	1	0.3714	0.1194	1	0.2773	0.1194	1	0.5275	0.0177	1	1.6842	0.0177	686.327	2896.779
2020	1	0.1675	0.0388	1	0.3611	0.1176	1	0.2701	0.1176	1	0.5184	0.0174	1	1.6544	0.0174	696.965	2941.679
2021	1	0.1645	0.0380	1	0.3560	0.1158	1	0.2667	0.1158	1	0.5094	0.0170	1	1.6250	0.0170	707.768	2987.275
2022	1	0.1621	0.0374	1	0.3521	0.1143	1	0.2643	0.1143	1	0.5025	0.0168	1	1.6023	0.0168	716.544	3024.318
2023	1	0.1597	0.0369	1	0.3483	0.1129	1	0.2618	0.1129	1	0.4956	0.0165	1	1.5799	0.0165	725.429	3061.819
2024	1	0.1574	0.0363	1	0.3444	0.1116	1	0.2594	0.1116	1	0.4889	0.0163	1	1.5578	0.0163	734.424	3099.786
2025	1	0.1551	0.0357	1	0.3407	0.1102	1	0.2570	0.1102	1	0.4823	0.0160	1	1.5361	0.0160	743.531	3138.223
2026	1	0.1529	0.0352	1	0.3374	0.1088	1	0.2550	0.1088	1	0.4757	0.0158	1	1.5146	0.0158	752.751	3177.137
2027	1	0.1508	0.0347	1	0.3342	0.1075	1	0.2530	0.1075	1	0.4692	0.0155	1	1.4934	0.0155	762.085	3216.534
2028	1	0.1487	0.0341	1	0.3310	0.1062	1	0.2511	0.1062	1	0.4628	0.0153	1	1.4726	0.0153	771.535	3256.419
2029	1	0.1466	0.0336	1	0.3279	0.1049	1	0.2491	0.1049	1	0.4565	0.0151	1	1.4520	0.0151	781.102	3296.798
2030	1	0.1446	0.0331	1	0.3247	0.1036	1	0.2472	0.1036	1	0.4503	0.0148	1	1.4317	0.0148	790.788	3337.678
2031	1	0.1425	0.0326	1	0.3210	0.1023	1	0.2447	0.1023	1	0.4438	0.0146	1	1.4099	0.0146	800.593	3379.066
2032	1	0.1397	0.0320	1	0.3161	0.1007	1	0.2414	0.1007	1	0.4354	0.0143	1	1.3823	0.0143	813.323	3432.793
2033	1	0.1371	0.0313	1	0.3114	0.0992	1	0.2382	0.0992	1	0.4272	0.0140	1	1.3554	0.0140	826.255	3487.374
2034	1	0.1345	0.0307	1	0.3067	0.0976	1	0.2350	0.0976	1	0.4191	0.0138	1	1.3289	0.0138	839.392	3542.823
2035	1	0.1319	0.0301	1	0.3021	0.0961	1	0.2319	0.0961	1	0.4112	0.0135	1	1.3030	0.0135	852.739	3599.154

Appendix D – Parking Charges

Calculation of Parking Charges

Parking charges are required as an input to the calculation of generalised cost for use in the Demand Model, with average cost per car used. These have been calculated for the major urban areas within the SEStran Regional Model area. Table D1 shows where these have been applied, and to which zones.

Table D1 Destination zones where parking charges are applied

City	SEStran Regional Model destination zone
Dunfermline	132
Edinburgh	218-223, 227,238, 239
Stirling	86-90

There are three types of parking charges in the model, these are:

- Long Stay Parking - this is appropriate for home based commute trips. We assume that 15% of total trips to the city centre for work are variations of 'kiss and ride' so no cost for parking is incurred, and also that 40% park in private non residential car parks. It is therefore assumed that 45% of total trips will pay this cost.

- Short Stay - this is applied to home based other and non-home based other trips. It is assumed that 80% of total trips will pay this charge; and

- No Charge – this is applied to Employers Business trips as it is perceived that even if they pay a parking charge they will not perceive the cost.

Average Charges

The average charges for each city are shown in Table D2. These are in 2007 prices and are the space weighted average car park charge over all car park sites in the area.

Table D2 Average charges

	Short Term (HO)	Long Term (HW)
Dunfermline	£1.26	£3.40
Edinburgh	£3.61	£9.14
Stirling	£1.74	£2.53

Application

These costs will be added to the base year generalised cost skim matrices after first being multiplied by the tolling parameter of the generalised cost equation. The costs will only be added to the zones defined in Table D1. Table D3 shows the calculated values for addition to the cost matrices. The values are also halved for input into the demand model (ie half of the cost is perceived in each direction)

Table D3 Parking Charges as Generalised Cost (Mins)

City	Short Term Cost	Long Term Cost
Dunfermline	8.05	12.26
Edinburgh	23.18	32.97
Stirling	11.17	9.11

(NB the long stay parking charge for Stirling in Generalised Minutes is less than the Short Term charge. This is due to the proportion of people paying the charge and the small differential between the two in monetary terms).

Appendix E – Software Versions

Software Versions

The following software versions have been used in the SEStran Regional Model:

- Demand Model – CUBE VOYAGER v5.0.2

- Highway Model – SATURN v10.8.17

- Demand Model Calibration – ALOGIT4 – v2.3

Appendix F – Incremental Adjustment Factors

Incremental Adjustment Factors

This appendix describes an approach for applying incremental factors to the assignment matrices for the SEStran Regional Model.

Typically in previous versions of CSTM3, TMfS etc we have applied the model incrementally by applying absolute models and using the outputs to modify a base 'observed' matrix. We have done this as follows (on a cell by cell basis):

B = base observed trips,

S_b = base modelled trips,

S_f = future modelled trips

F = future trips

Then:

$$F = B + S_f - S_b$$

This formula has been applied to all cells of the matrix. In other projects we have used a multiplicative approach rather than additive, eg:

$$F = B * S_f / S_b$$

This method is based on the assumption that in both base and future years all modelled cells in the matrix will be non zero, which is likely to be the case so long as we don't have zero K-factors in the destination choice model. Also if the base year is zero then so will all of the future years.

Both methods would probably be fine if the differences between the Base **B** and the modelled S_b are reasonable (except for the zero **B** case in the multiplicative). However we have never looked closely at the comparisons between the base and the modelled previously and this would be a time consuming process to start now. It is more appropriate to devise a process which is a mixture of increments and factors as appropriate. The aims of the process are to ensure that zero **B** case is taken into account and that the changes in trips are in keeping with the scale of B and that we avoid applying large growth factors base on small values of S_f and S_b to cells with large **B**.

We can define five cases:

- zero **B**;
- low **B**, high S_b;
- low **B** low S_b
- high **B** high S_b;

- high **B** low S_b.

For (1) the forecasts trips will be S_f minus S_b. The growth factor method does not work here.

For (2), (3) and (4) we use the multiplicative form. In the case of (2) use of the incremental could add a large amount to a cell which has relatively small number of base trips. For (3) and (4) it could make little overall difference if we used incremental or multiplicative.

For (5) we should use the increments method. This is the case where relatively large growth factors based on small modelled flows could create excessive growth when applied to high base.

Each cell in the matrix will need to be identified as being increments or multiplicative. The key will be to identify cases (5). We can define and agree a critical value of B/S_b such that values above this will be considered as case (5) and those below as cases (2), (3) and (4).

Appendix G – Park and Ride Site Files

Park and Ride Sites

Table G1 below shows all the Park and Ride sites included in the SEStran Regional base model and the capacity of the Main car park at each of these sites. Note, those sites in italics are bus based park and ride sites

Table G1 Park and Ride Sites

Local Authority	Park and Ride Sites	Zone	Modelled Car Park Capacity
City of Edinburgh	Brunstane	377	0
	Curriehill	383	40
	Dalmeny	385	140
	Hermiston	399	495
	Ingilston	400	1080
	Newcraighall	417	600
	South Gyle	424	0
East Lothian	Drem	386	78
	Dunbar	387	89
	Longniddry	414	76
	Musselburgh	416	125
	North Berwick	418	99
	Prestonpans	421	176
	Wallyford	428	421
Falkirk	Camelon	379	28
	Falkirk Grahamston	392	380
	Falkirk High	393	282
	Larbert	408	285

	Polmont	420	188
	Aberdour	371	94
	Burntisland	378	19
	Cardenden	380	13
	Cowdenbeath	381	138
	Cupar	382	70
	Dalgety Bay	384	194
	Dunfermline Queen Margaret	388	86
	Dunfermline Town	389	265
	Ferrytoll	396	1040
	Glenrothes	397	48
	Inverkeithing	401	425
	Kincardine	402	67
	Kinghorn	403	0
	Kirkcaldy	405	633
	Ladybank	407	20
	Leuchars	409	159
	Lochgelly	413	13
	Markinch	415	148
	North Queensferry	419	13
	Rosyth	422	135
Fife	Springfield	425	0
	Addiewell	373	12
	Bathgate	374	185
	Breich	375	0
West Lothian	Fauldhouse	395	9

Kirknewton	406	30
Linlithgow	410	189
Livingston North	411	166
Livingston South	412	120
Uphall	427	140
West Calder	377	27

**Note this is based on a combination of NRTS data and data obtained from the SEStran Regional Park and Ride Strategy, to give an idea of the number of spaces currently being used outside the official station car parks. The number of cars using 'Far' spaces can actually exceed this value.

Appendix H – Park and Ride Model Calibration

Park and Ride Calibration

Table H1 below shows the calibration of the Park and Ride Module by site

Table H1 Park and Ride Calibration by Site

Site Name	Total Observed	Total Modelled	Difference	GEH
Aberdour	9	11	2	0.54
Bathgate	367	362	-5	0.24
Bridge of Allan	30	32	2	0.45
Brunstane	40	71	30	4.08
Burntisland	23	24	1	0.25
Camelon	27	42	15	2.53
Cupar	10	4	-7	2.56
Curriehill	0	7	7	3.66
Dalgety Bay	128	109	-19	1.77
Dalmeny	95	96	1	0.07
Dunblane	23	7	-16	4.14
Dunfermline Queen Margaret	40	37	-3	0.49
Dunfermline Town	71	76	4	0.52
Edinburgh Park	53	31	-22	3.35
Falkirk Grahamston	24	61	38	5.77
Ferrytoll	394	*454*	*60*	*2.90*
Gleneagles	0	0	0	0.00
Glenrothes	13	16	3	0.66
Haymarket	14	11	-3	0.98
Hermiston	345	*366*	*21*	*1.13*
Ingilston	419	*426*	*7*	*0.33*
Inverkeithing	543	521	-22	0.96
Kirkcaldy	379	328	-52	2.76

Kirknewton	18	12	-6	1.62
Ladybank	15	2	-12	4.17
Larbert	69	83	14	1.57
Leuchars	28	8	-19	4.53
Linlithgow	303	266	-37	2.19
Livingston North	185	173	-13	0.95
Livingston South	64	107	43	4.60
Markinch	41	47	6	0.94
Musselburgh	0	11	11	4.56
North Queensferry	20	17	-3	0.72
Newcraighall	*36*	*62*	*27*	*3.78*
Polmont	188	186	-2	0.15
Rosyth	22	28	6	1.13
South Gyle	8	24	16	4.14
Springkerse	*200*	*226*	*25*	*1.75*
Stirling	74	74	0	0.03
Uphall	72	59	-13	1.66
Wallyford	6	10	3	1.14
Waverley	2	0	-2	2.06
Wester Hailes	0	58	58	10.73
Falkirk High	367	318	-49	2.64
Total	**4769**	**4861**	**93**	**1.34**

Appendix I – Other Model Parameters

Occupancy Factors

The occupancy factors for the Base modelled year are shown in table I1. The occupancy factors have been developed by applying growth factors to those used in the TMfS:05A, which came from RSI data. Over time these values change in line with guidance from WebTAG (Unit 3.5.6).

Table I1 Base Year Occupancies by Sector and Purpose

Purpose / Direction	AM	IP	PM
Home Based Employers Business	1.16	1.19	1.19
Home Based Work	1.18	1.14	1.15
Home Based Other	1.48	1.60	1.67
Home Based Education	1.86	1.62	1.75

Period to Hour Factors

The Period to Hour factors used in the SEStran Regional Model are as follows:

- AM Peak – 0.38 for Highway and 0.5 for PT;

- Inter Peak – 1/6 for Highway and PT; and

- PM Peak – 0.36 for Highway and 0.42 for PT.

These factors are applied to all user classes and all purposes. The Peak PT factors are greater than the Highway as they have more 'peaky' peak hours.

PM Factors

In the Model the PM *From-Home* matrices are derived by factoring the Inter-Peak *From-Home* matrices. The factors used are shown in table I2 below. The factors are applied to the entire matrices.

Table I2 Inter Peak to PM *From-Home* Factors

	C11C, C12C,C2C	*C11P,C12P,C2P*	*C0P*
HE	0.286	0.167	0.200
HO	0.355	0.122	0.067
HW	0.226	0.158	0.244

Reverse Factors – To Home and Non Home Based

The Factors used in the Reverse process to go from *From-Home* trips to *To-Home* trips are shown in table I3. The factors to create *Non-Home Based* Productions from *From-Home* trips are shown in table I4. The factors to create *Non-Home Based* Attractions from *To-Home* trips are shown in table I4.

It should be noted that C1 in the tables below means car is available these factors are then applied to C11, C12 and C2 household segments.

Table I3 To Home Factors

			hbw am car	hbw am pt 1+	hbw am pt 0	hbeb am car	hbeb am pt 1+	hbeb am pt 0	hbo am car	hbo am pt 1+	hbo am pt 0	hbw inter car	hbw inter pt 1+	hbw inter pt 0	hbeb inter car	hbeb inter pt 1+	hbeb inter pt 0	hbo inter car	hbo inter pt 1+	hbo inter pt 0	hbw pm car	hbw pm pt 1+	hbw pm pt 0	hbeb pm car	hbeb pm pt 1+	hbeb pm pt 0	hbo pm car	hbo pm pt 1+	hbo pm pt 0
hbw	am	car	0.009	0.001	0.000	0.000	0.000	0.000	0.001	0.000	0.000	0.244	0.003	0.001	0.003	0.001	0.000	0.021	0.000	0.000	0.689	0.007	0.001	0.011	0.000	0.000	0.054	0.001	0.000
hbw	am	pt 1+	0.000	0.006	0.000	0.000	0.000	0.000	0.000	0.003	0.000	0.018	0.169	0.000	0.000	0.000	0.000	0.012	0.015	0.000	0.065	0.653	0.000	0.003	0.006	0.000	0.024	0.009	0.000
hbw	am	pt 0	0.000	0.000	0.017	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.183	0.000	0.000	0.004	0.000	0.000	0.022	0.070	0.000	0.652	0.000	0.000	0.004	0.013	0.000	0.022
hbeb	am	car	0.000	0.000	0.000	0.031	0.000	0.000	0.009	0.000	0.000	0.050	0.000	0.000	0.255	0.003	0.000	0.075	0.000	0.000	0.069	0.000	0.000	0.453	0.000	0.000	0.075	0.000	0.000
hbeb	am	pt 1+	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.300	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.500	0.000	0.000	0.000	0.000
hbeb	am	pt 0	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.500	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
hbo	am	car	0.002	0.000	0.000	0.001	0.000	0.000	0.164	0.000	0.000	0.018	0.001	0.000	0.004	0.000	0.000	0.495	0.005	0.002	0.054	0.000	0.000	0.005	0.000	0.000	0.246	0.005	0.001
hbo	am	pt 1+	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.005	0.000	0.000	0.000	0.005	0.000	0.024	0.490	0.000	0.014	0.033	0.000	0.000	0.010	0.000	0.038	0.362	0.000
hbo	am	pt 0	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.048	0.000	0.000	0.020	0.000	0.000	0.004	0.040	0.000	0.611	0.004	0.000	0.016	0.000	0.000	0.000	0.024	0.000	0.210
hbw	inter	car	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.146	0.000	0.000	0.002	0.000	0.000	0.019	0.000	0.000	0.412	0.000	0.000	0.004	0.000	0.000	0.042	0.000	0.000
hbw	inter	pt 1+	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.132	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.132	0.184	0.000	0.000	0.000	0.000	0.026	0.000	0.000
hbw	inter	pt 0	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.122	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.341	0.000	0.000	0.000	0.000	0.000	0.000
hbeb	inter	car	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.013	0.000	0.000	0.383	0.000	0.000	0.071	0.000	0.000	0.026	0.000	0.000	0.364	0.000	0.000	0.071	0.000	0.000
hbeb	inter	pt 1+	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.667	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.167	0.000
hbeb	inter	pt 0	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.200	0.000	0.000	0.000	0.000	0.000
hbo	inter	car	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.001	0.000	0.000	0.000	0.000	0.000	0.631	0.003	0.001	0.003	0.000	0.000	0.001	0.000	0.000	0.314	0.002	0.001
hbo	inter	pt 1+	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.003	0.000	0.000	0.000	0.000	0.051	0.520	0.000	0.000	0.003	0.000	0.000	0.000	0.000	0.044	0.297	0.000
hbo	inter	pt 0	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.001	0.000	0.000	0.000	0.013	0.000	0.635	0.000	0.000	0.001	0.000	0.000	0.000	0.020	0.000	0.298
hbw	pm	car	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.075	0.000	0.000	0.000	0.000	0.000	0.028	0.000	0.000
hbw	pm	pt 1+	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
hbw	pm	pt 0	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
hbeb	pm	car	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.023	0.000	0.000	0.205	0.000	0.000	0.000	0.000	0.000
hbeb	pm	pt 1+	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
hbeb	pm	pt 0	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
hbo	pm	car	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.003	0.000	0.000	0.002	0.000	0.000	0.384	0.001	0.001
hbo	pm	pt 1+	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.028	0.167	0.000
hbo	pm	pt 0	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.020	0.000	0.196

Table I2 Production Factors for Non Home Based Trips

			nhbeb am car	nhbeb am pt 1+	nhbeb am pt 0	nhbo am car	nhbo am pt 1+	nhbo am pt 0	nhbeb inter car	nhbeb inter pt 1+	nhbeb inter pt 0	nhbo inter car	nhbo inter pt 1+	nhbo inter pt 0	nhbeb pm car	nhbeb pm pt 1+	nhbeb pm pt 0	nhbo pm car	nhbo pm pt 1+	nhbo pm pt 0
hbw	am	car	0.027	0.001	0.000	0.006	0.000	0.000	0.035	0.001	0.000	0.044	0.001	0.000	0.002	0.000	0.000	0.041	0.001	0.000
hbw	am	pt 1+	0.003	0.005	0.000	0.000	0.003	0.000	0.003	0.005	0.000	0.010	0.018	0.000	0.000	0.000	0.000	0.016	0.021	0.000
hbw	am	pt 0	0.000	0.000	0.011	0.000	0.000	0.000	0.007	0.000	0.000	0.007	0.000	0.036	0.000	0.000	0.000	0.007	0.000	0.040
hbeb	am	car	0.042	0.000	0.000	0.031	0.000	0.000	0.092	0.008	0.000	0.034	0.000	0.000	0.003	0.000	0.000	0.005	0.000	0.000
hbeb	am	pt 1+	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.200	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
hbeb	am	pt 0	0.000	0.000	0.500	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
hbo	am	car	0.006	0.000	0.000	0.063	0.002	0.000	0.003	0.000	0.000	0.057	0.001	0.000	0.001	0.000	0.000	0.005	0.000	0.000
hbo	am	pt 1+	0.000	0.000	0.000	0.000	0.033	0.000	0.004	0.000	0.000	0.017	0.066	0.000	0.000	0.000	0.000	0.000	0.017	0.000
hbo	am	pt 0	0.000	0.000	0.003	0.000	0.000	0.052	0.003	0.000	0.000	0.003	0.000	0.081	0.000	0.000	0.000	0.000	0.000	0.007
hbw	inter	car	0.000	0.000	0.000	0.000	0.000	0.000	0.032	0.004	0.000	0.087	0.002	0.000	0.010	0.000	0.000	0.028	0.000	0.000
hbw	inter	pt 1+	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.026	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.026	0.000
hbw	inter	pt 0	0.000	0.000	0.000	0.000	0.000	0.000	0.023	0.000	0.000	0.000	0.000	0.068	0.000	0.000	0.000	0.000	0.000	0.023
hbeb	inter	car	0.000	0.000	0.000	0.000	0.000	0.000	0.103	0.000	0.000	0.058	0.000	0.000	0.032	0.000	0.000	0.006	0.000	0.000
hbeb	inter	pt 1+	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.143	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
hbeb	inter	pt 0	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
hbo	inter	car	0.000	0.000	0.000	0.000	0.000	0.000	0.001	0.000	0.000	0.119	0.002	0.000	0.001	0.000	0.000	0.019	0.000	0.000
hbo	inter	pt 1+	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.009	0.053	0.000	0.000	0.000	0.000	0.000	0.025	0.000
hbo	inter	pt 0	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.003	0.000	0.072	0.000	0.000	0.000	0.000	0.000	0.012
hbw	pm	car	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.009	0.000	0.000	0.097	0.000	0.000
hbw	pm	pt 1+	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
hbw	pm	pt 0	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
hbeb	pm	car	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.068	0.000	0.000	0.000	0.000	0.000
hbeb	pm	pt 1+	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
hbeb	pm	pt 0	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1.000	0.000	0.000
hbo	pm	car	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.001	0.000	0.000	0.044	0.000	0.000
hbo	pm	pt 1+	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.024	0.024	0.000
hbo	pm	pt 0	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.018	0.000	0.000

Table I3 Attraction Factors for Non Home Based Trips

			nhbeb am car	nhbeb am pt 1+	nhbeb am pt 0	nhbo am car	nhbo am pt 1+	nhbo am pt 0	nhbeb inter car	nhbeb inter pt 1+	nhbeb inter pt 0	nhbo inter car	nhbo inter pt 1+	nhbo inter pt 0	nhbeb pm car	nhbeb pm pt 1+	nhbeb pm pt 0	nhbo pm car	nhbo pm pt 1+	nhbo pm pt 0
hbw	am	car	0.000	0.000	0.000	0.385	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
hbw	am	pt 1+	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
hbw	am	pt 0	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
hbeb	am	car	0.182	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
hbeb	am	pt 1+	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
hbeb	am	pt 0	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
hbo	am	car	0.000	0.000	0.000	0.068	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
hbo	am	pt 1+	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
hbo	am	pt 0	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
hbw	inter	car	0.003	0.000	0.000	0.024	0.000	0.000	0.044	0.000	0.000	0.028	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
hbw	inter	pt 1+	0.000	0.000	0.000	0.000	0.017	0.000	0.000	0.034	0.000	0.000	0.017	0.000	0.000	0.000	0.000	0.000	0.000	0.000
hbw	inter	pt 0	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.024	0.000	0.000	0.098	0.000	0.000	0.000	0.000	0.000	0.000
hbeb	inter	car	0.047	0.000	0.000	0.000	0.000	0.000	0.154	0.007	0.000	0.020	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
hbeb	inter	pt 1+	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.300	0.000	0.100	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
hbeb	inter	pt 0	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
hbo	inter	car	0.001	0.000	0.000	0.009	0.000	0.000	0.004	0.000	0.000	0.140	0.003	0.000	0.000	0.000	0.000	0.000	0.000	0.000
hbo	inter	pt 1+	0.000	0.000	0.000	0.000	0.003	0.000	0.000	0.000	0.000	0.007	0.070	0.000	0.000	0.000	0.000	0.000	0.000	0.000
hbo	inter	pt 0	0.000	0.000	0.000	0.000	0.000	0.005	0.000	0.000	0.000	0.003	0.000	0.096	0.000	0.000	0.000	0.000	0.000	0.000
hbw	pm	car	0.002	0.000	0.000	0.009	0.000	0.000	0.041	0.000	0.000	0.041	0.000	0.000	0.026	0.001	0.000	0.010	0.000	0.000
hbw	pm	pt 1+	0.004	0.000	0.000	0.000	0.004	0.000	0.004	0.004	0.000	0.000	0.012	0.000	0.000	0.000	0.000	0.000	0.004	0.000
hbw	pm	pt 0	0.000	0.000	0.000	0.000	0.000	0.006	0.018	0.000	0.006	0.000	0.000	0.018	0.000	0.000	0.000	0.000	0.000	0.000
hbeb	pm	car	0.044	0.000	0.000	0.000	0.000	0.000	0.151	0.004	0.000	0.012	0.000	0.000	0.119	0.008	0.000	0.012	0.000	0.000
hbeb	pm	pt 1+	0.000	0.200	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
hbeb	pm	pt 0	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.500	0.000	0.000	0.000
hbo	pm	car	0.000	0.000	0.000	0.003	0.000	0.000	0.006	0.000	0.000	0.093	0.002	0.001	0.006	0.000	0.000	0.179	0.007	0.002
hbo	pm	pt 1+	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.016	0.078	0.000	0.000	0.000	0.000	0.010	0.094	0.000
hbo	pm	pt 0	0.000	0.000	0.003	0.000	0.000	0.003	0.000	0.000	0.000	0.000	0.000	0.090	0.000	0.000	0.000	0.003	0.000	0.093

Appendix J – High Occupancy Vehicle Modelling

Overview

This appendix describes the implementation of High Occupancy Vehicles (HOV) modelling in the model. It explains the changes required to the network to undertake HOV modelling and the implementation of occupancy choice in the demand model.

Network

To undertake HOV assignments an additional links, with the HOV only link type, must be coded into the networks. Single Occupant Vehicles (SOV) are then unable to use these links.

The public transport network, and particularly the PT lines file, must be adjusted to incorporate the modifications made to the Highway network. The main issue is to keep the node sequence in the lines file consistent with the Highway network where new nodes were introduced.

Tolls must also be adjusted were appropriate to reflect costs on the HOV lane. In most cases the HOV lane toll will be zero. The tolls on the existing links can remain as they were or be changed as appropriate.

Demand Model

The demand model outputs an assignment matrix with the normal purposes, but with each of the car purposes split into SOV and HOV. The output assignment matrix has the following tables:

- Car In Work SOV;
- Car In Work HOV;
- Car Commute SOV;
- Car Commute HOV;
- Car Other SOV;
- Car Other HOV;
- LGV; and
- HGV.

After the assignment stage the SOV purposes and the HOV purposes are skimmed separately. These are then combined for input into the trip frequency, time of day choice mode and destination choice stages (These stages are not affected by the implementation of HOV Modelling). There is a separate stage below about how the combination of HOV and SOV costs is undertaken.

At the end of the inner loops, the Occupancy Choice module has been introduced to split the *From-Home* demand matrices between SO and HO person matrices.

The occupancy choice takes the form of a logit model, using different generalised costs for Single Occupant (SO) and High Occupancy (HO) person trips.

The person trip proportion of single occupancy for a particular *ij* pair will be:

$$P_{so} = exp(ß * C_{SO})/\{ exp(ß * C_{SO}) + exp(ß * (C_{HO} + ∂))\}$$

Where,

C_{SO} is the generalised cost for Single Occupancy and C_{HO} is the generalised cost for High Occupancy.

ß is a sensitivity parameter and *∂* is a high occupancy penalty representing the additional travel and difficulty in arranging passengers.

∂ is calibrated to match observed HOV and SOV splits. For forecast years, *∂* can be calculated from the forecast year Reference Case runs.

The next step is the calculation of *To-Home* and *Non-Home Based* matrices for AM peak and Inter-Peak periods, and *From-Home*, *To-Home* and *Non-Home Based* matrices for the PM peak period. This is undertaken through the normal reverse processing method, where the same reverse factors are applied to HOV trips and SOV trips.

The person peak period are then reduced from peak period to peak hour and converted from person to vehicle using HOV Occupancy Matrices (SOV occupancy is 1). The HOV occupancy matrices are calculated from the average occupancy matrices previously used in the model using the SOV / HOV split.

Then, incremental adjustments are applied. These are not split between SOV and HOV. They apply to the whole car vehicle matrices. The SOV / HOV split before the incremental adjustments is saved and applied to the post incremental adjustments car matrix.

No change occurs to the creation of the PT matrices.

Cost Skimming / Generalised Cost Calculation

The generalised cost matrices are skimmed from the Highway and PT assignments. However with the HOV modelling, the Highway cost skimming produces separate cost matrices for SOV and HOV.

The two generalised cost matrices for Car SOV and Car HOV need to be combined to get a unique Generalised Cost matrix, as required for the mode split and the destination choice for the start of the next external demand model loop.

SOV and HOV cost were initially combined to calculate an average cost. However, during initial testing this average cost resulted in an increase for both HOV and SOV demands. The increase in the SOV flow was quite significant and not in line with expectations.

After investigation, it was clear that the use of average cost was the source of the problem. Therefore, it was decided to use change in composite cost instead of average cost, as explained below.

For average costs:

$$C_{AV} = C_{HO} + P_{SO} * (C_{SO} - C_{HO})$$ (1)

For composite cost:

$$C_{COMP} = C_{SO} - (\ln P_{SO} / \beta)$$ (2)

For both equations P_{SO} is the person proportion for single occupancy not the vehicle proportion.

Instead of using the average cost calculation we can use the change in composite costs between 'base' and 'scheme' to add to the base costs for input to the destination choice. Using formula (2) above for the low occupancy only:

$$\Delta C_{COMP} = C_{SCHEME} - C_{BASE} + \ln (P_{BASE} / P_{SCHEME}) / \beta$$

Adding in C_{BASE} this then reduces to:

$$C = C_{SCHEME} + \ln (P_{BASE} / P_{SCHEME}) / \beta$$ (3)

For O/D movements which are not in scope for the scheme is being tested then equation (3) reduces to C_{SCHEME} which is the standard cost for both low and high occupancy. So there is no need to make any corrections to the calculation, it will apply to all movements.

In addition all of the information required for equation (3) is calculated anyway in the high/low occupancy allocation, so no new calculations are required.

HOV Modelling Feature

The HOV modelling new feature has no significant impact on model results when no specific HOV lane is introduced (ie no network change). This means the demand matrices created by the demand model – with or without HOV modelling feature – will be similar when adding SOV and HOV car matrices together.

Parameter Calibration

The parameters used in the HOV Model for commuters are derived from a stated preference survey, for which the outcomes are contained in separate report (TMfS High Occupancy Vehicle Lane: Stated Preference Research, MVA Consultancy, September 2008). The parameters for the other purposes will be defined as the model is used.

Appendix K – Macro Time of Day Choice Modelling

Macro Time of Day Choice

Macro Time of Day (MTOD) choice relates to changing the time period of travel in response to relative changes in the costs of travel between time periods.

Methodology

In terms of the methodology for implementation of MTOD, including the conditions for when it is recommended for inclusion in the demand model, the position in the model hierarchy and the choice of parameter values, the Guidance on Variable Demand Modelling published in WebTAG unit 3.10 was used. The key points in the guidance are:

- Paragraph 1.8.3 of TAG Unit 3.10.3 says, "Macro time period choice should be considered when strong cost differentials between time periods are expected to develop or change." In particular the introduction of differential charges for road use or public transport between peak and inter peak time periods; and

- Paragraph 1.11.17 of TAG Unit 3.10.3 says, "when long time periods, of the order of three hours are being modelled, macro time period choice should be positioned either just before or just after main mode choice in the model hierarchy with parameter values similar in magnitude to the main mode choice parameter values."

Given that there has been little practical experience on implementing this choice process it was deemed appropriate to take on board this guidance in developing MTOD in the SEStran Regional Model, since this guidance was developed taking on board all currently available research on the topic.

Implementation

The implementation of MTOD has the following features:

- It has been implemented before main mode choice in the model choice hierarchy, which is consistent with the guidance;

- The logsum composite costs are used in the MTOD choice process. The implication of this is that changes in relative road congestion between time periods will also influence time period choice. The reason for this is that average tolls by time period cannot be used alone for MTOD choice because it is possible that average toll paid per vehicle in the inter peak could be larger than that for the morning peak even though the basic toll rates are lower in the inter peak;

- The scaling parameter will need to lie between 0.0 and 1.0 and preferably close to 1.0 in order to be consistent with the guidance. The value initially input into the model is 0.8, however, this may be refined when testing a live application;

- It has been included in the demand model as an optional feature. It is recommended that it is used only for model applications where there is the introduction of differential charging for road use between time periods, which is also consistent with the guidance;

- It is implemented for the morning peak period only since it applies only to 'From-Home' trips, which are not modelled explicitly in the Evening Peak; and

- The process generates a pre-peak matrix for the morning peak. The pre-peak matrix creates 'To-Home' in the other time periods, using the Morning Peak reverse factors. The process is done by journey purpose and at the person trip stage. The model is applied separately for each row in the matrix.

The model is based on an incremental logit formulation. For the morning peak we have the model formulae:

P_{pre}, P_{peak}, P_{inter} are the base proportions in each time period. Note that we do not have P_{pre} so an estimate is made and it is set to 0.2;

P'_{pre}, P'_{peak}, P'_{inter} are the forecast proportions in each time period.

We define $A = P_{inter} / P_{peak}$, and $B = P_{pre} / P_{peak}$

We also have ΔC_{pre}, ΔC_{peak} and ΔC_{inter} as the generalised cost skim differences from the base in the do something for the three time periods.

Where, $\Delta C = C' - C$

The change in costs in the pre-peak (ΔC_{pre}) is assumed to be zero, as there is very little or no congestion in the Pre-Peak time period

Then:

$P'_{peak} = 1 / (1 + A*exp(\beta*(\Delta C_{inter} - \Delta C_{peak})) + B*exp(\beta*(\Delta C_{pre} - \Delta C_{peak})))$

$P'_{inter} = 1 / (1 + (1/A)*exp(\beta*(\Delta C_{peak} - \Delta C_{inter})) + (B/A)*exp(\beta*(\Delta C_{pre} - \Delta C_{inter})))$

$P'_{pre} = 1 - P'_{peak} - P'_{inter}$

The base proportions of 'From-Home' trips in the Morning Peak and Inter-Peak were calculated from model data at the 'From-Home' trip end level, along with the assumption that $P_{pre}=0.2$.

The position of MTOD within the model hierarchy is shown in Appendix A.

Appendix L – Peak Spreading Modelling

Peak Spreading

The peak spreading implemented in the model is intended to cover supply led Peak Spreading, where increased congestion forces travellers to change their departure time to avoid the worst congestion.

The Peak Spreading module in the SEStran Regional Model is an incremental logit model, but operates at the matrix cell level rather than for the whole matrix. It is therefore responsive to schemes and policies as well as taking into account the effects of overall traffic growth.

Given the run time penalties which would be incurred if additional shoulder peak assignments were to be carried out, the generalised costs for the shoulder peak hours are approximated based on the peak hour costs. It should be noted that the assumption is made that the flow levels in the two shoulder peaks are equal.

The basis of the peak spreading model is that at each outer loop in the demand model we find an approximation of the peak spreading supply/demand equilibrium. This is undertaken by making use of linear approximations to the supply and demand functions.

The demand function can be written as:

$$P_{forecast} = P_{base} / [P_{base} + \{ (1 - P_{base}) * (exp ([\beta * \Delta C]) \}]$$

where $\Delta C = (C'_{shoulder} - C'_{peak}) - (C_{shoulder} - C_{peak})$ (C is the cost in the base and C' is the forecast cost);

P_{base} is the base peak period to hour factor;

P_{base} is the forecast peak period to hour factor; and

β is the sensitivity parameter.

This can be re-written as:

$$P_{forecast} = 1 / [1 + \{ ([1/P_{base}]-1) * (exp ([\beta * \Delta C]) \}\}$$

A linear approximation of the form $P_{forecast} = P_{base} + slope * \Delta C$ can be developed by calculating the slope between two extremes of interest for $P_{forecast}$. If we use the range 0.3 to 0.6 for our extremes then we get the formula:

$$slope = (0.6-0.3) * \beta / (ln ((1/0.6)-1) - ln ((1/0.3)-1))$$

$$= 0.3 * \beta / (ln (0.6667/2.3333))$$

$$= - 0.2315 * \beta$$

The prediction of $P_{forecast}$ using the linear formula was compared with the prediction using the full formula for a range β values. The results showed the linear formula to be a good

representation for the range of $P_{forecast}$ considered. The results are independent of the value of β, the value of which determines the sensitivity to ΔC.

For the supply side formulae we have to make an assumption that a linear approximation is valid for our needs. The supply formula has to show the relationship between the peak proportion and the associated cost difference (ΔC) on the assumption of a fixed peak period demand.

Shoulder peak costs are estimated by a procedure whereby:

- link flows from a converged assignment are reduced by a percentage which represents the *average* ratio of shoulder peak to peak flows;

- link journey times are modified to be consistent with the reduced flows; and

- minimum costs are then skimmed from the network.

This method then gives an estimate of the shoulder peak costs for each cell in the matrix. It is however an approximation since we have to use an average flow reduction rather than cell specific reductions. It was judged however to be a suitable basis for estimating supply functions for each matrix cell.

The method for estimating shoulder costs therefore gives us an estimate of ΔC at each outer loop of the model for each cell in the matrix. We therefore know one point on the supply curve. This point is (P_n, ΔC_n) where n is the number of the outer loop.

There is also another point on the supply curve, which is known. When $P_{forecast} = 1/3$ then $(C'_{shoulder} - C'_{peak}) = 0$. Therefore $\Delta C = -(C_{shoulder} - C_{peak})$. For convenience this can be called ΔC_{base}. The second point therefore is $(1/3, \Delta C_{base})$.

The supply relationship can be defined as:

$P = \boldsymbol{a}_s + (\boldsymbol{b}_s * \Delta C)$

Where $\boldsymbol{a}_s$ and $\boldsymbol{b}_s$ are parameters.

From the two known points we have:

$0.333 = \boldsymbol{a}_s + (\boldsymbol{b}_s * \Delta C_{base})$

$P_n = \boldsymbol{a}_s + (\boldsymbol{b}_s * \Delta C_n)$

From which we calculate

$\boldsymbol{b}_s = (P_n - 0.333)/(\Delta C_n - \Delta C_{base})$

$\boldsymbol{a}_s = 0.333 - (\boldsymbol{b}_s * \Delta C_{base})$

We can express the linear approximation to the demand relationship as:

$P = \boldsymbol{a}_d + (\boldsymbol{b}_d * \Delta C)$

where $\boldsymbol{a}_d = P_{base}$ and $\boldsymbol{b}_d = -0.2315 * \beta$

The estimated equilibrium position is the point at which these two lines meet:

$$P'_n = a_d + (b_d * (a_s - a_d)/(b_d - b_s))$$

This process is shown diagrammatically in Figure L1. The supply and demand lines have been labelled SS and DD but the diagram is not meant to be in strict economics format.
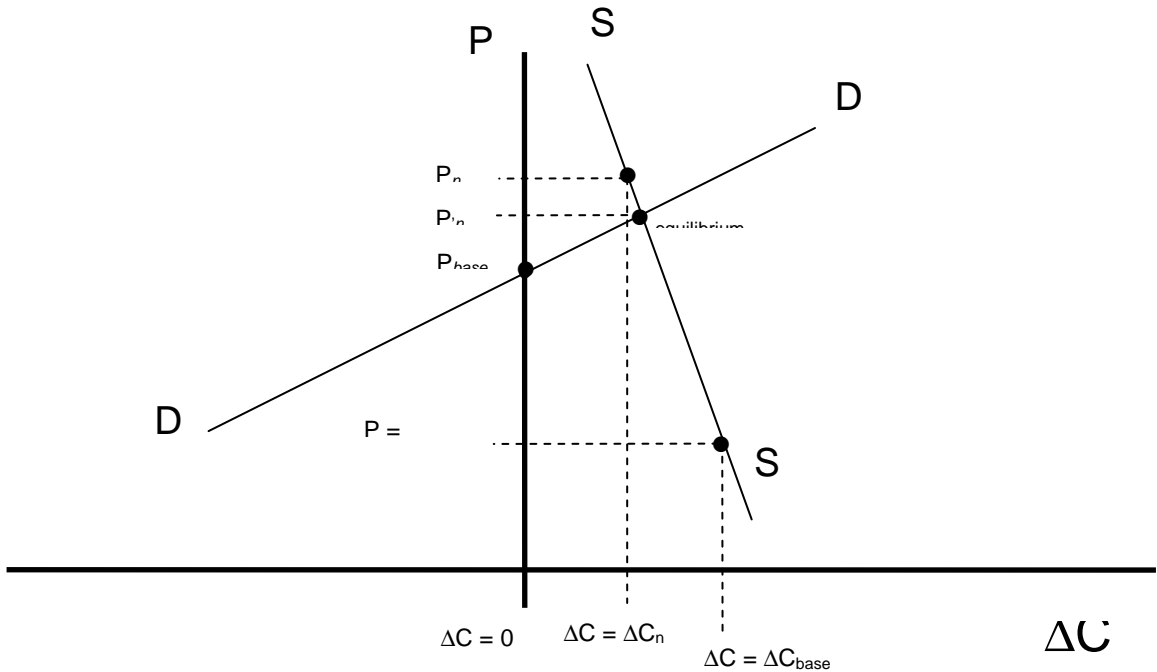

Figure L1 Peak Spreading Equilibrium Process

The value of ΔC_n has been calculated (as described in 8.2.8) for each cell in the matrix but using a flow reduction factor based on the average value of the ratio of shoulder to peak flows. An adjustment to ΔC_n can be made to give a better estimate for each matrix cell. The adjustment is:

$$\Delta C_n = \Delta C_{n(average)} *(1 - a_{ij})/(1-Av)$$

Where a_{ij} is the ratio of shoulder to peak flow for cell ij

Av is the average ratio of shoulder to peak flow for the whole matrix